Appendix 12: Vector Calculus

The del operator

$$\nabla = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z} \tag{A12.1}$$

The divergence

$$\nabla \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \tag{A12.2}$$

The gradient

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z \tag{A12.3}$$

The Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{A12.4}$$

The Laplacian in cylindrical coordinates

$$x = r\cos q \ , \ y = r\sin q \ , \ z = z \tag{A12.6}$$

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial q^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

$$\nabla^2 \Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial q^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

The Laplacian in spherical coordinates

$$x = r\sin q\cos f \ , \ x = r\sin q\sin f \ , \ z = r\cos q \tag{A12.7}$$

$$\nabla^2 \Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial \Psi}{\partial r}\right) + \frac{1}{r^2\sin q}\frac{\partial}{\partial q}\left(\sin q\frac{\partial \Psi}{\partial q}\right) + \frac{1}{r^2\sin^2 q}\frac{\partial^2 \Psi}{\partial f^2}$$

The vector product

$$\vec{\nabla} x \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \tag{A12.5}$$

$$= (\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z})\,\vec{e}_x + (\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z})\,\vec{e}_y + (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y})\,\vec{e}_z$$