4.9. LEDs

A light emitting diode consists of a p-n diode, which is designed so that radiative recombination dominates. Homojunction p-n diodes, heterojunction p-i-n diodes where the intrinsic layer has a smaller bandgap (this structure is also referred to as a double-hetero-structure) and p-n diodes with a quantum well in the middle are all used for LEDs. We will only consider the p-n diode with a quantum well because the analytical analysis is more straightforward and also since this structure is used often in LEDs and even more frequently in laser diodes.

4.9.1. Rate equations

The LED rate equations are derived from the continuity equations as applied to the p-n diode:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - R + G \tag{4.9.1}$$

where G is the generation rate per unit volume and R is the recombination rate per unit volume. This equation is now simplified by integrating in the direction perpendicular to the plane of the junction. We separate the integral in two parts: one for the quantum well, one for the rest of the structure.

$$\int_{\text{qw}}\frac{\partial n}{\partial t} + \int_{\text{p-n}}\frac{\partial n}{\partial t} = \frac{J}{q} - \frac{J_{SHR}}{q} - \frac{J_{bb}}{q} - \frac{J_{ideal}}{q} - \sum_k (N_k P_k - N_{ik}^2)B_k - \frac{NP - N_{i1}^2}{N + P + 2N_{i1}}\frac{1}{t_{nr}} \tag{4.9.2}$$

where k refers to the quantum number in the well. If we ignore the carriers everywhere except in the quantum well and assume that only the first quantum level is populated with electrons/holes and that the density of electrons equals the density of holes, we obtain:

$$\frac{\partial N}{\partial t} = \frac{J}{q} - B_1 N^2 - \frac{N}{2t_{nr}} + \frac{S}{t_{ab}} \tag{4.9.3}$$

where the last term is added to include reabsorption of photons. The rate equation for the photon density including loss of photons due to emission (as described with the photon lifetime t_{ph}) and absorption (as described with the photon absorption time t_{ab}) equals:

$$\frac{\partial S}{\partial t} = B_1 N^2 - \frac{S}{t_{ph}} + \frac{S}{t_{ab}} \tag{4.9.4}$$

The corresponding voltage across the diode equals:

$$V_a = \frac{E_{g,qw}}{q} + V_t \ln[(e^{N/N_c} - 1)(e^{N/N_{V*}} - 1)] \tag{4.9.5}$$

Where the modified effective hole density of states in the quantum well, N_v*, accounts for the occupation of multiple hole levels as described in section 4.4.3.d. The optical output power is given by the number of photons, which leave the semiconductor per unit time, multiplied with the photon energy:

$$P_{out} = hn \frac{S}{t_{ph}} A(1-R)\frac{\Theta_c^2}{4} \tag{4.9.6}$$

where A is the active area of the device, R is the reflectivity at the surface and Θ_c is the critical angle for total internal reflection[1]

$$R = (\frac{n_1 - n_2}{n_1 + n_2})^2 \text{ and } \Theta_c = \sin^{-1}(\frac{n_1}{n_2}) \tag{4.9.7}$$

The reflectivity and critical angle for a GaAs Air interface are 30 % and 16^o respectively.

4.9.2. DC solution to the rate equations

The time independent solution in the absence of reabsorption, as indicated with the subscript 0, is obtained from:

$$0 = \frac{J_0}{q} - BN_0^2 - \frac{N_0}{2t_{ph}} \tag{4.9.8}$$

$$0 = BN_0^2 - \frac{S_0}{t_{ph}} \tag{4.9.9}$$

where B is the bimolecular recombination constant. Solving these equations yields:

$$N_0 = \frac{1}{4Bt_{nr}}[\sqrt{1 + \frac{16t_{nr}^2 BJ_0}{q}} - 1] \tag{4.9.10}$$

for small currents this reduces to: ($J << q/16t_{nr}^2 B$)

$$N_0 = \frac{2t_{nr} J_0}{q} \tag{4.9.11}$$

which indicates that SHR recombination dominates, whereas for large currents one finds: ($J >> q/16t_{nr}^2 B$)

$$N_0 = \sqrt{\frac{J_0}{qB}} \tag{4.9.12}$$

The DC optical output power is:

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[1] See Appendix A.7 for the derivation of the reflectivity at dielectric interfaces.

$$P_0 = hn\ BN_0^2 A(1 - R) \tag{4.9.13}$$

This expression explains the poor efficiency of an LED. Even if no non-radiative recombination occurs in the active region of the LED, most photons are confined to the semiconductor because of the small critical angle. Typically only a few percent of the photons generated escape the semiconductor. This problem is most severe for planar surface emitting LEDs. Better efficiencies have been obtained for edge emitting, "superluminescent" LEDs (where stimulated emission provides a larger fraction of photons which can escape the semiconductor) and LEDs with curved surfaces.

4.9.3. AC solution to the rate equations

Assume that all variables can be written as a sum of a time independent term and a time dependent term (note that $n(t)$ is still a density per unit area):

$$N = N_0 + n_1(t) \quad J = J_0 + j_1(t) \tag{4.9.14}$$

$$S = S_0 + s_1(t) \quad P = P_0 + p_1(t) \tag{4.9.15}$$

$$V_a = V_{a,0} + v_a(t) \tag{4.9.16}$$

The rate equations for the time dependent terms the given by:

$$\frac{\partial n_1}{\partial t} = -B2N_0 n_1(t) - Bn_1^2(t) - \frac{n_1(t)}{2t_0} + \frac{j_1(t)}{q} \tag{4.9.17}$$

Needs beta ?? $$\frac{\partial s_1}{\partial t} = B2N_0 n_1(t) + Bn_1^2(t) - \frac{s_1(t)}{t_{ph}} \tag{4.9.18}$$

Assuming the AC current of the form $j_1 = j_{1,0}e^{jwt}$ and ignoring the higher order terms we can obtain a harmonic solution of the form:

$$n_1 = n_{1,0}e^{jwt} \quad s_1 = s_{1,0}e^{jwt} \quad p_1 = p_{1,0}e^{jwt} \tag{4.9.19}$$

yielding:

$$s_{1,0} = \frac{1}{q}\frac{B2N_0 j_{1,0} t_{ph} t_{eff}}{(1 + jwt_{ph})(1 + jwt_{eff})} \tag{4.9.20}$$

where t_{eff} depends on N_0 as:

$$t_{eff} = \frac{1}{2BN_0 + \frac{1}{2t_0}} \quad (4.9.21)$$

and the AC responsivity is:

$$\frac{p_{1,0}}{j_{1,0}} = \frac{hn}{q}(1-R)\frac{p\Theta_c^2}{4p}\frac{B2N_0 t_{eff}}{(1+jwt_{ph})(1+jwt_{eff})} \quad (4.9.22)$$

at $w = 0$ this also yields the differential quantum efficiency (D.Q.E)

$$\text{D.Q.E.} = \frac{p_{1,0}}{j_{1,0}}\frac{q}{hn} = \frac{(1-R)\Theta_c^2 BN_0 t_{eff}}{2} = \frac{(1-R)\Theta_c^2 BN_0 t_0}{4BN_0 t_0 + 1} \quad (4.9.23)$$

4.9.4. Equivalent circuit of an LED

The equivalent circuit of an LED consists of the p-n diode current source parallel to the diode capacitance and in series with a linear series resistance, *R*. The capacitance, *C*, is obtained from:

$$\frac{1}{C} = \frac{dV_a}{dQ} = \frac{dV_a}{qdN} \quad (4.9.24)$$

$$\frac{1}{C} = \frac{1}{q}V_t \frac{[\frac{1}{N_c}e^{N/N_c}(e^{N/N_v}-1) + \frac{1}{N_v}e^{N/N_v}(e^{N/N_c}-1)]}{(\mathrm{e}^{N/N_c}-1)(\mathrm{e}^{N/N_v}-1)} \quad (4.9.25)$$

or

$$C = \frac{qN_0}{mV_t} \quad (4.9.26)$$

with

$$m = \frac{N_0 e^{N/N_c}}{N_c(e^{N/N_c}-1)} + \frac{N_0 e^{N/N_v}}{N_v(e^{N/N_v}-1)} \quad (4.9.27)$$

for $N_0 << N_c$ and/or N_v, $m = 2$ while for $N_0 >> N_c$ and/or N_v, $m = \frac{N_0(N_c + N_v)}{N_c N_v}$