Chapter 6: MOS Capacitors

6.3. MOS analysis

$$V_{FB} = \Phi_M - \Phi_S \tag{6.3.1}$$

$$\Phi_M - \Phi_S = \Phi_M - c - \frac{E_g}{q} - V_t \ln(\frac{N_a}{n_i}) \tag{6.3.2}$$

$$\Phi_{poly,S} = \Phi_{poly} - \Phi_S = V_t \ln(\frac{N_{a,poly}}{N_a}) \quad \text{p - type polysilicon gate} \tag{6.3.3}$$

$$\Phi_{poly,S} = \Phi_{poly} - \Phi_S = V_t \ln(\frac{n_i^2}{N_{d,poly} N_a}) \quad \text{n - type polysilicon gate}$$

$$V_{FB} = \Phi_{MS} - \frac{Q_i}{C_{ox}} - \frac{1}{e_{ox}} \int_0^{t_{ox}} r_{ox}(x)\, x\, dx \tag{6.3.4}$$

$$\begin{aligned} Q_{inv} &= C_{ox}(V_G - V_T) & &\text{for} & V_G &> V_T \\ Q_{inv} &= 0 & & & V_G &\le V_T \end{aligned} \tag{6.3.5}$$

$$Q_d = -qN_a x_d \tag{6.3.6}$$

$$E_s = \frac{qN_a x_d}{e_s} \text{ and } E_{ox} = \frac{qN_a x_d}{e_{ox}} \tag{6.3.7}$$

$$f_s = \frac{qN_a x_d^2}{2e_s} \tag{6.3.8}$$

$$f_F = V_t \ln \frac{N_a}{n_i} \tag{6.3.9}$$

$$x_d = \sqrt{\frac{2e_s f_s}{qN_a}}, \text{for } 0 \le f_s \le 2f_F \tag{6.3.10}$$

$$Q_{d,T} = -qN_a x_{d,T} \tag{6.3.11}$$

$$x_{d,T} = \sqrt{\frac{2e_s(2f_F)}{qN_a}} \tag{6.3.12}$$

$$Q_M = -(Q_d + Q_{inv}) \tag{6.3.13}$$

$$V_G = V_{FB} + f_s + \frac{Q_M}{C_{ox}} = V_{FB} + f_s - \frac{Q_d + Q_{inv}}{C_{ox}} \tag{6.3.14}$$

$$V_G = V_{FB} + f_s + \frac{\sqrt{2e_s qN_a f_s}}{C_{ox}}, \text{ for } 0 \le f_s \le 2f_F \tag{6.3.15}$$

$$V_G = V_{FB} + 2f_F + \frac{\sqrt{4e_s qN_a f_F}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}} \overset{\Delta}{=} V_T - \frac{Q_{inv}}{C_{ox}} \tag{6.3.16}$$

$$V_T = V_{FB} + 2f_F + \frac{\sqrt{4e_s qN_a f_F}}{C_{ox}} \tag{6.3.17}$$

$$C_{LF} = C_{HF} = C_{ox}, \text{ for } V_G \le V_{FB} \tag{6.3.18}$$

$$C_{LF} = C_{HF} = \frac{1}{\frac{1}{C_{ox}} + \frac{x_d}{e_s}}, \text{ for } V_{FB} \le V_G \le V_T \tag{6.3.19}$$

$$x_d = \sqrt{\frac{2e_s f_s}{qN_d}} \tag{6.3.20}$$

$$V_G = V_{FB} + f_s + \frac{\sqrt{2e_s qN_a f_s}}{C_{ox}}, \text{ for } 0 \le f_s \le 2f_F \tag{6.3.15}$$

$$C_{LF} = C_{ox} \text{ and } C_{HF} = \frac{1}{\frac{1}{C_{ox}} + \frac{x_{d,T}}{e_s}}, \text{ for } V_G \ge V_T \tag{6.3.21}$$

$$\frac{d^2 f}{dx^2} = \frac{q}{e_s}(N_a^+ - p) = \frac{qN_a}{e_s}(1 - e^{-\frac{f}{V_t}}) \cong \frac{qN_a}{e_s}\frac{f}{V_t} \tag{6.3.22}$$

$$f = f_s e^{-\frac{x}{L_D}}, \text{ with } L_D = \sqrt{\frac{e_s V_t}{qN_a}} \tag{6.3.23}$$

$$C_{s,FB} = \frac{dQ_s}{df_s} = \frac{d(e_s \frac{f_s}{L_D})}{df_s} = \frac{e_s}{L_D} \tag{6.3.24}$$

$$C_{FB} = \frac{1}{\frac{1}{C_{ox}} + \frac{L_D}{e_s}} \tag{6.3.25}$$

$$time = \frac{|Q_{inv}|}{qG(x_{d,dd} + L_n)} = \frac{C_{ox}(V_G - V_T)}{\frac{qn_i}{2t}\left(\sqrt{\frac{2e_s f_{s,dd}}{qN_a}} + \sqrt{m_n V_t t_n}\right)} \tag{6.3.26}$$

$$\frac{dV_G}{dt} > \frac{qn_i}{2C_{ox}}\sqrt{\frac{m_n V_t}{t_n}} \tag{6.3.27}$$