A16. Appendix 16: Equation Sheet

Chapter 1: Review of Modern Physics

1.2. Quantum mechanics

$$l = \frac{h}{p} \qquad E_{ph} = hn = \hbar w = \frac{hc}{l}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x)$$

$$E_n = \frac{h^2}{2m^*}\left(\frac{n}{2L_x}\right)^2, \text{ with } n = 1, 2, \ldots \qquad E_n = -\frac{m_0 q^4}{8e_0^2 h^2 n^2}, \text{ with } n = 1, 2, \ldots$$

1.3. Electromagnetic theory

$$\frac{d\mathcal{E}(x)}{dx} = \frac{r(x)}{e} \qquad \frac{df(x)}{dx} = -\mathcal{E}(x)$$

$$\frac{d^2 f(x)}{dx^2} = -\frac{r(x)}{e}$$

Chapter 2: Semiconductor Fundamentals

2.3. Energy bands

$$E_g(T) = E_g(0) - \frac{aT^2}{T+b}$$

2.4. Density of states

$$g_c(E) = \frac{1}{L^3}\frac{dN}{dE} = \frac{8p\sqrt{2}}{h^3} m^{*3/2}\sqrt{E-E_c}\text{, for } E \geq E_c$$

$$g_c(E) = 0\text{, for } E < E_c$$

2.5. Carrier distribution functions

$$f(E) = \frac{1}{1+\mathrm{e}^{(E-E_F)/kT}}$$

$$f_{donor}(E_d) = \frac{1}{1+\frac{1}{2}\mathrm{e}^{(E_d-E_F)/kT}} \qquad f_{acceptor}(E_a) = \frac{1}{1+4\mathrm{e}^{(E_a-E_F)/kT}}$$

$$f_{BE}(E) = \frac{1}{\mathrm{e}^{(E-E_F)/kT}-1} \qquad f_{MB}(E) = \frac{1}{\mathrm{e}^{(E-E_F)/kT}}$$

2.6. Carrier densities

$$n_o = \int_{E_c}^{\infty} g_c(E) f(E) dE \qquad p_o = \int_{-\infty}^{E_v} g_v(E)[1-f(E)]dE$$

$$n_o = \frac{2}{3}\frac{\sqrt{2}}{p^2}\left(\frac{m^*}{\hbar^2}\right)^{3/2}(E_F - E_c)^{3/2}\text{, for } E_F \geq E_c \text{ and } T = 0 \text{ K}$$

$$n_o = N_c e^{\frac{E_F - E_c}{kT}} \text{ with } N_c = 2[\frac{2p\, m_e^* kT}{h^2}]^{3/2}$$

$$p_o = N_v e^{\frac{E_v - E_F}{kT}} \text{ with } N_v = 2[\frac{2p\, m_h^* kT}{h^2}]^{3/2}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

$$n_o \cdot p_o = n_i^2$$

$$E_i = \frac{E_c + E_v}{2} + \frac{1}{2} kT \ln(\frac{N_v}{N_c}) = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln(\frac{m_h^*}{m_e^*})$$

$$n_o = n_i e^{(E_F - E_i)/kT}$$

$$p_o = n_i e^{(E_i - E_F)/kT}$$

$$E_F = E_i + kT \ln \frac{n_o}{n_i}$$

$$E_F = E_i - kT \ln \frac{p_o}{n_i}$$

$$E_c - E_d = 13.6 \frac{m_{cond}^*}{m_0 e_r^2} \text{ eV}$$

$$n_o = \frac{N_d^+ - N_a^-}{2} + \sqrt{(\frac{N_d^+ - N_a^-}{2})^2 + n_i^2}$$

$$p_o = \frac{N_a^- - N_d^+}{2} + \sqrt{(\frac{N_a^- - N_d^+}{2})^2 + n_i^2}$$

$$n = n_o + d\, n = n_i \exp(\frac{F_n - E_i}{kT})$$

$$p = p_o + d\, p = n_i \exp(\frac{E_i - F_p}{kT})$$

2.7. Carrier Transport

$$m \overset{\Delta}{=} \frac{|\vec{v}|}{|\vec{E}|} = \frac{qt}{m}$$

$$m = m_{\min} + \frac{m_{\max} - m_{\min}}{1 + (\frac{N}{N_r})^a}$$

$$\vec{J} = qnm_n \vec{E}$$

$$J = qnv_e + qpv_h = q(nm_n + pm_p)E$$

$$s \overset{\Delta}{=} \frac{J}{E} = q(nm_n + pm_p)$$

$$r = \frac{1}{s} = \frac{1}{q(m_n n + m_p p)}$$

$$v(E) = \frac{mE}{1 + \frac{mE}{v_{sat}}}$$

$$J_n = qD_n \frac{dn}{dx}$$

$$J_p = -qD_p \frac{dp}{dx}$$

$$D_n = m_n \frac{kT}{q} = m_n V_t$$

$$D_p = m_p \frac{kT}{q} = m_p V_t$$

$$J_n = qnm_n E + qD_n \frac{dn}{dx}$$

$$J_p = qpm_p E - qD_p \frac{dp}{dx}$$

$$I_{total} = A(J_n + J_p)$$

2.8. Carrier recombination and generation

$$U_n = R_n - G_n = \frac{n_p - n_{p0}}{t_n}$$

$$U_p = R_p - G_p = \frac{p_n - p_{n0}}{t_p}$$

$$U_{b-b} = b(np - n_i^2)$$

$$U_{SHR} = \frac{pn - n_i^2}{p + n + 2n_i \cosh(\frac{E_i - E_t}{kT})} N_t v_{th} s$$

$$U_{Auger} = \Gamma_n n(np - n_i^2) + \Gamma_p p(np - n_i^2)$$

$$G_{p,light} = G_{n,light} = a \frac{P_{opt}(x)}{E_{ph} A}$$

$$\frac{dP_{opt}(x)}{dx} = -aP_{opt}(x)$$

2.9. Continuity equation

$$\frac{\P\, n(x,t)}{\P\, t} = \frac{1}{q} \frac{\partial J_n(x,t)}{\partial x} + G_n(x,t) - R_n(x,t)$$

$$\frac{\P\, p(x,t)}{\P\, t} = -\frac{1}{q} \frac{\partial J_p(x,t)}{\partial x} + G_p(x,t) - R_p(x,t)$$

$$\frac{\P\, n(x,t)}{\P\, t} = D_n \frac{\P^{\,2} n_p(x,t)}{\P\, x^2} - \frac{n_p(x,t) - n_{p0}}{t_n} \qquad \frac{\P\, p(x,t)}{\P\, t} = D_p \frac{\P^{\,2} p_n(x,t)}{\P\, x^2} - \frac{p_n(x,t) - p_{n0}}{t_p}$$

$$0 = D_n \frac{d^2 n_p(x)}{dx^2} - \frac{n_p(x) - n_{p0}}{t_n} \qquad 0 = D_p \frac{d^2 p_n(x)}{dx^2} - \frac{p_n(x) - p_{n0}}{t_p}$$

2.10. The drift-diffusion model

$$r = q(p - n + N_d^+ - N_a^-) \qquad \frac{dE}{dx} = \frac{r}{e}$$

$$\frac{df}{dx} = -E \qquad \frac{dE_i}{dx} = qE$$

$$n = n_i e^{(F_n - E_i)/kT} \qquad p = n_i e^{(E_i - F_p)/kT}$$

$$J_n = qnm_n E + qD_n \frac{dn}{dx} \qquad J_p = qpm_p E - qD_p \frac{dp}{dx}$$

$$0 = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh(\frac{E_t - E_i}{kT})}\frac{1}{t} \qquad 0 = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh(\frac{E_t - E_i}{kT})}\frac{1}{t}$$

Chapter 3: Metal-Semiconductor Junctions

3.2. Structure and principle of operation

$f_B = \Phi_M - c$, for an n-type semiconductor

$$f_B = \frac{E_g}{q} + c - \Phi_M$$, for a p-type semiconductor

$$f_i = \Phi_M - c - \frac{E_c - E_{F,n}}{q} \quad , \quad \text{n - type} \qquad\qquad f_i = c + \frac{E_c - E_{F,p}}{q} - \Phi_M \quad , \quad \text{p - type}$$

3.3. Electrostatic analysis

$$\frac{d^2 f}{dx^2} = -\frac{r}{e_s} = -\frac{q}{e_s}(p - n + N_d^+ - N_a^-)$$

$$\frac{d^2 f}{dx^2} = \frac{2qn_i}{e_s}(\sinh\frac{f - f_F}{V_t} + \sinh\frac{f_F}{V_t}) \qquad\qquad \sinh\frac{f_F}{V_t} = \frac{N_a^- - N_d^+}{2n_i}$$

$$r(x) = qN_d \qquad 0 < x < x_d$$
$$r(x) = 0 \qquad x_d < x$$

$$E(x) = -\frac{qN_d}{e_s}(x_d - x) \qquad 0 < x < x_d$$
$$E(x) = 0 \qquad x_d \le x$$

$$f(x) = 0 \qquad x \le 0$$
$$f(x) = \frac{qN_d}{2e_s}[x_d^2 - (x_d - x)^2] \qquad 0 < x < x_d$$
$$f(x) = \frac{qN_d x_d^2}{2e_s} \qquad x_d \le x$$

$$E(x=0) = -\frac{qN_d x_d}{e_s} = -\frac{Q_d}{e_s} \qquad\qquad f_i - V_a = -f(x=0) = \frac{qN_d x_d^2}{2e_s}$$

$$x_d = \sqrt{\frac{2e_s(f_i - V_a)}{qN_d}} \qquad\qquad C_j = \left|\frac{dQ_d}{dV_a}\right| = \sqrt{\frac{qe_s N_d}{2(f_i - V_a)}} = \frac{e_s}{x_d}$$

3.4. Schottky diode current

$$J_n = qm_n E_{\max} N_c \exp(-\frac{f_B}{V_t})[\exp(\frac{V_a}{V_t}) - 1] \qquad\qquad E_{\max} = \sqrt{\frac{2q(f_i - V_a)N_d}{e_s}}$$

$$J_n = qv_R N_c \exp(-\frac{f_B}{V_t})[\exp(\frac{V_a}{V_t}) - 1] \qquad v_R = \sqrt{\frac{kT}{2p\ m}}$$

$$J_n = q\, v_R\, n\, \Theta \qquad \Theta = \exp\left(-\frac{4}{3}\frac{\sqrt{2qm^*}}{\hbar}\frac{f_B{}^{3/2}}{E}\right)$$

Chapter 4: p-n Junctions

4.2. Structure and principle of operation

$$f_i = V_t \ln \frac{N_d\, N_a}{n_i^2} \qquad f = f_i - V_a$$

4.3. Electrostatic analysis of a p-n diode

$$x_d = x_n + x_p$$

$$r = q(p - n + N_d^+ - N_a^-) \cong q(N_d^+ - N_a^-), \text{ for } - x_p \le x \le x_n$$

$$\begin{aligned}
r(x) &= 0 && \text{, for } x \le -x_p \\
r(x) &= -qN_a && \text{, for } -x_p \le x \le 0 \\
r(x) &= qN_d && \text{, for } 0 \le x \le x_n \\
r(x) &= 0 && \text{, for } x_n \le x
\end{aligned}$$

$$Q_n = qN_d x_n \qquad Q_p = -qN_a x_p$$

$$\frac{dE(x)}{dx} = \frac{r(x)}{e_s} \cong \frac{q}{e_s}(N_d^+(x) - N_a^-(x)), \text{ for } -x_p \le x \le x_n$$

$$E(x) = 0 \quad \text{, for } x \le -x_p$$

$$E(x) = -\frac{qN_a(x + x_p)}{e_s} \quad \text{, for } -x_p \le x \le 0$$

$$E(x) = \frac{qN_d(x - x_n)}{e_s} \quad \text{, for } 0 \le x \le x_n$$

$$E(x) = 0 \quad \text{, for } x_n \le x$$

$$E(x = 0) = -\frac{qN_a x_p}{e_s} = -\frac{qN_d x_n}{e_s} \qquad E(x = 0) = -\frac{2(f_i - V_a)}{x_d}$$

$$N_d x_n = N_a x_p$$

$$x_n = x_d \frac{N_a}{N_a + N_d} \qquad x_p = x_d \frac{N_d}{N_a + N_d}$$

$$f_i - V_a = \frac{qN_d x_n^2}{2e_s} + \frac{qN_a x_p^2}{2e_s} \qquad x_d = \sqrt{\frac{2e_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(f_i - V_a)}$$

$$x_n = \sqrt{\frac{2e_s}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}(f_i - V_a)} \qquad x_p = \sqrt{\frac{2e_s}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}(f_i - V_a)}$$

$$C_j = \sqrt{\frac{qe_s}{2(f_i - V_a)}\frac{N_a N_d}{N_a + N_d}} \qquad C_j = \frac{e_s}{x_d}$$

4.4. The p-n diode current

$$p_n(x = x_n) = p_{n0} e^{V_a / V_t} \qquad n_p(x = -x_p) = n_{p0} e^{V_a / V_t}$$

$$p_n(x \ge x_n) = p_{n0} + Ae^{-(x - x_n)/L_p} + Be^{(x - x_n)/L_p}$$

$$n_p(x \le -x_p) = n_{p0} + Ce^{-(x + x_p)/L_p} + De^{(x + x_p)/L_p}$$

$$p_n(x \ge x_n) = p_{n0} + p_{n0}(e^{V_a/V_t} - 1)[\cosh\frac{x - x_n}{L_p} - \coth\frac{w_n^{'}}{L_p}\sinh\frac{x - x_n}{L_p}]$$

$$n_p(x \le -x_p) = n_{p0} + n_{p0}(e^{V_a/V_t} - 1)[\cosh\frac{x + x_p}{L_n} + \coth\frac{w_p^{'}}{L_n}\sinh\frac{x + x_p}{L_n}]$$

$$w_n^{'} = w_n - x_n \qquad w_p^{'} = w_p - x_p$$

$$I = A[J_n(x = -x_p) + J_p(x = x_n) + J_r] \cong I_s(e^{V_a/V_t} - 1)$$

$$I_s = qA[\frac{D_n n_{p0}}{L_n}\coth(\frac{w_p^{'}}{L_n}) + \frac{D_p p_{n0}}{L_p}\coth(\frac{w_n^{'}}{L_p})] \text{ (general case)}$$

$$I_s = qA[\frac{D_n n_{p0}}{L_n} + \frac{D_p p_{n0}}{L_p}] \text{ (“long” diode)} \qquad I_s = qA[\frac{D_n n_{p0}}{w_p^{'}} + \frac{D_p p_{n0}}{w_n^{'}}] \text{ (“short” diode)}$$

$$J_{b-b} = qn_i^2 bw(e^{V_a/V_t} - 1) \qquad J_{SHR} = \frac{qn_i x^{'}}{2t}(e^{V_a/2V_t} - 1)$$

$$V_a^* = V_a + IR_s$$

$$J = J_s e^{V_a/hV_t} \qquad h = \frac{\log(e)}{V_t \text{ slope}} = \frac{\frac{1}{\text{slope}}}{59.6 \text{ mV/decade}}$$

$$\Delta Q_p = I_{s,p}(e^{V_a/V_t} - 1)t_p \qquad I_{s,p} = q\frac{A p_{n0} D_p}{L_p}$$

$$C_{d,p} = \frac{I_{s,p} e^{V_a/V_t} t_p}{V_t} \text{ (“long” diode)}$$

$$C_{d,p} = \frac{I_{s,p} e^{V_a/V_t} t_{r,p}}{V_t} \text{ (“short” diode)} \qquad \text{with } t_{r,p} = \frac{w_p^{'2}}{2D_p}$$

4.5. Reverse bias breakdown

$$|V_{br}| = -f_i + \frac{|E_{br}|^2 e_s}{2qN}$$

$$w_{br} = \frac{|E_{br}| e_s}{qN}$$

$$M = \frac{1}{1 - \left|\dfrac{V_a}{V_{br}}\right|^n}, \text{ where } 2 < n < 6$$

$$J_n = q v_R\, n\, \Theta$$

$$\Theta = \exp\left(-\frac{4}{3}\frac{\sqrt{2m^*}}{q\hbar}\frac{E_g^{3/2}}{E}\right)$$

4.6. Optoelectronic devices

$$I = I_s(e^{V_a/V_t} - 1) - I_{ph}$$

$$I_{ph,\max} = \frac{q}{hn} P_{in}$$

$$I_{ph} = (1-R)(1-e^{-a\,d})\frac{qP_{in}}{hn}$$

$$< i^2 >= 2\, q\, I\, \Delta f$$

$$\text{Fill Factor} = \frac{I_m V_m}{I_{sc} V_{oc}}$$

$$\text{Roundtrip amplification} = e^{2gL} R_1 R_2 = 1$$

$$g = \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

$$P_{out} = h \frac{hn}{q}(I - I_{th})$$

Chapter 5: Bipolar Junction Transistors

5.2. Structure and principle of operation

$$w_E^{'} = w_E - x_{n,BE}$$

$$w_B^{'} = w_B - x_{p,BE} - x_{p,BC}$$

$$w_C^{'} = w_C - x_{n,BC}$$

$$x_{n,BE} = \sqrt{\frac{2e_s(f_{i,BE} - V_{BE})}{q}\frac{N_B}{N_E}\left(\frac{1}{N_B + N_E}\right)}$$

$$x_{p,BE} = \sqrt{\frac{2e_s(f_{i,BE} - V_{BE})}{q}\frac{N_E}{N_B}\left(\frac{1}{N_B + N_E}\right)}$$

$$x_{p,BC} = \sqrt{\frac{2e_s(f_{i,BC} - V_{BC})}{q}\frac{N_C}{N_B}\left(\frac{1}{N_B + N_C}\right)}$$

$$x_{n,BC} = \sqrt{\frac{2e_s(f_{i,BC} - V_{BC})}{q}\frac{N_B}{N_C}\left(\frac{1}{N_B + N_C}\right)}$$

$$f_{i,BE} = V_t \ln \frac{N_B N_E}{n_i^2}$$

$$f_{i,BC} = V_t \ln \frac{N_B N_C}{n_i^2}$$

$$I_E = I_C + I_B$$

$$I_E = I_{E,n} + I_{E,p} + I_{r,d}$$

$$I_C = I_{E,n} - I_{r,B}$$

$$I_B = I_{E,p} + I_{r,B} + I_{r,d}$$

$$a = \frac{I_C}{I_E}$$

$$b = \frac{I_C}{I_B} = \frac{a}{1-a}$$

$$a = a_T\, g_E\, d_r$$

$$g_E = \frac{I_{E.n}}{I_{E,n} + I_{E,p}}$$

$$a_T = \frac{I_{E,n} - I_{r,B}}{I_{E,n}}$$

$$d_r = \frac{I_E - I_{r,d}}{I_E}$$

5.3. Ideal transistor model

$$I_{E,n} = qn_i^2 A_E \left(\frac{D_{n,B}}{N_B w_B^{'}} \right) \left(\exp(\frac{V_{BE}}{V_t}) - 1 \right) \qquad I_{E,p} = qn_i^2 A_E \left(\frac{D_{p,E}}{N_E w_E^{'}} \right) \left(\exp(\frac{V_{BE}}{V_t}) - 1 \right)$$

$$\Delta Q_{n,B} = q\, A_E \, \frac{n_i^2}{N_B} \left(\exp(\frac{V_{BE}}{V_t}) - 1 \right) \frac{w_B^{'}}{2} \qquad I_{r,B} = \frac{\Delta Q_{n,B}}{t_n}$$

$$I_{E,n} = \frac{\Delta Q_{n,B}}{t_r} \qquad t_r = \frac{w_B^{'\,2}}{2D_{n,B}}$$

$$g_E = \frac{1}{1 + \frac{D_{p,E} N_B w_B^{'}}{D_{n,B} N_E w_E^{'}}} \qquad a_T = 1 - \frac{t_r}{t_n} = 1 - \frac{w_B^{'\,2}}{2D_{n,B} t_n}$$

$$b \cong \frac{D_{n,B} N_E w_E^{'}}{D_{p,E} N_B w_B^{'}}, \quad \text{if} \quad a \cong g_E$$

$$|V_A| = \frac{Q_B}{C_{j,BC}} = \frac{qN_B w_B^{'}}{\frac{e_s}{x_{p,BC} + x_{n,BC}}} \qquad n = \frac{1}{V_t \frac{d \ln I_C}{dV_{BE}}} \cong 1 + \frac{V_t}{Q_B} C_{j,BE}$$

Chapter 6: MOS Capacitors

6.3 MOS analysis

$$V_{FB} = \Phi_M - \Phi_S$$

$$\Phi_M - \Phi_S = \Phi_M - c - \frac{E_g}{2q} - V_t \ln(\frac{N_a}{n_i}) \text{ (nMOS)} \quad \Phi_M - \Phi_S = \Phi_M - c - \frac{E_g}{2q} + V_t \ln(\frac{N_d}{n_i}) \text{ (pMOS)}$$

$$\Phi_{poly} - \Phi_S = V_t \ln(\frac{N_{a,poly}}{N_a}) \qquad \text{p - type polysilico n gate}$$

$$\Phi_{poly} - \Phi_S = V_t \ln(\frac{n_i^2}{N_{d,poly} N_a}) \qquad \text{n - type polysilico n gate}$$

$$V_{FB} = \Phi_{MS} - \frac{Q_i}{C_{ox}} - \frac{1}{e_{ox}} \int_0^{t_{ox}} r_{ox}(x)\, x\, dx \qquad \text{with } C_{ox} = \frac{e_{ox}}{t_{ox}}$$

$$\begin{matrix} Q_{inv} = C_{ox}(V_G - V_T) & \text{for} & V_G > V_T \\ Q_{inv} = 0 & & V_G \le V_T \end{matrix}$$

$$f_F = V_t \ln \frac{N_a}{n_i} \text{ (nMOS)} \qquad f_F = V_t \ln \frac{N_d}{n_i} \text{ (pMOS)}$$

$$x_d = \sqrt{\frac{2 e_s f_s}{q N_a}}, \text{for } 0 \le f_s \le 2 f_F \qquad x_{d,T} = \sqrt{\frac{4 e_s f_F}{q N_a}}$$

$$V_T = V_{FB} + 2 f_F + \frac{\sqrt{4 e_s q N_a f_F}}{C_{ox}} \text{ (nMOS)} \qquad V_T = V_{FB} - 2 f_F - \frac{\sqrt{4 e_s q N_d f_F}}{C_{ox}} \text{ (pMOS)}$$

$$C_{LF} = C_{HF} = C_{ox}, \text{for } V_G \le V_{FB} \qquad C_{LF} = C_{HF} = \frac{1}{\frac{1}{C_{ox}} + \frac{x_d}{e_s}}, \text{for } V_{FB} \le V_G \le V_T$$

$$C_{LF} = C_{ox} \text{ and } C_{HF} = \frac{1}{\frac{1}{C_{ox}} + \frac{x_{d,T}}{e_s}}, \text{ for } V_G \geq V_T$$

$$C_{FB} = \frac{1}{\frac{1}{C_{ox}} + \frac{L_D}{e_s}} \qquad \text{with } L_D = \sqrt{\frac{e_s V_t}{q N_a}}$$

Chapter 7: MOS Field Effect Transistors

7.3. MOSFET analysis

Linear model

$$I_D = mC_{ox}\frac{W}{L}(V_{GS} - V_T)V_{DS}, \text{ for } |V_{DS}| << (V_{GS} - V_T)$$

Quadratic model

$$I_D = mC_{ox}\frac{W}{L}[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}], \quad \text{for } V_{DS} < V_{GS} - V_T$$

$$I_{D,sat} = mC_{ox}\frac{W}{L}\frac{(V_{GS} - V_T)^2}{2}, \quad \text{for } V_{DS} > V_{GS} - V_T$$

$$g_{m,quad} = mC_{ox}\frac{W}{L}V_{DS} \qquad g_{m,sat} = mC_{ox}\frac{W}{L}(V_{GS} - V_T)$$

$$g_{d,quad} = mC_{ox}\frac{W}{L}(V_{GS} - V_T - V_{DS}) \qquad g_{d,sat} = 0$$

Channel length modulation

$$I_{D,sat} = \mu C_{ox} \frac{W}{L} \frac{(V_{GS} - V_T)^2}{2} (1 + \lambda V_{DS}), \quad \text{for } V_{DS} > V_{GS} - V_T$$

Variable depletion layer model

$$I_D = \frac{\mu_n C_{ox} W}{L} (V_{GS} - V_{FB} - 2\phi_F - \frac{V_{DS}}{2}) V_{DS}$$
$$- \frac{2}{3} \mu_n \frac{W}{L} \sqrt{2\varepsilon_s q N_a} ((2\phi_F + V_{DB})^{3/2} - (2\phi_F + V_{SB})^{3/2})$$

$$V_{DS,sat} = V_{GS} - V_{FB} - 2\phi_F - \frac{qN_a \varepsilon_s}{C_{ox}^2} \{\sqrt{1 + 2\frac{C_{ox}^2}{qN_a \varepsilon_s}(V_{GB} - V_{FB})} - 1\}$$

$$g_{m,sat} = \mu_n^* C_{ox} \frac{W}{L} (V_{GS} - V_T) \qquad \mu_n^* = \mu_n \left(1 - \frac{1}{\sqrt{1 + \frac{2(2\phi_F + V_{SB}) C_{ox}^2}{qN_a \varepsilon_s}}} \right)$$

7.4. Threshold voltage

$$\Delta V_T = \gamma (\sqrt{(2\phi_F + V_{SB})} - \sqrt{2\phi_F}) \qquad \gamma = \frac{\sqrt{2\varepsilon_s q N_a}}{C_{ox}}$$

7.7. Advanced MOSFET issues

$$I_D \propto \exp(\frac{V_G - V_T}{V_t})$$

$$\mu_{surface} \propto \mathcal{E}^{-1/3}$$