

## Problems

1. Calculate the packing density of the body centered cubic, the face centered cubic and the diamond lattice, listed in example 2.1.
2. At what temperature does the energy bandgap of silicon equal exactly 1 eV?
3. Consider within a close-packed FCC crystal, the largest space between the atoms. How many atoms surround this space and what is the size of the largest sphere that can be placed between those atoms. Assume the atoms to be hard rigid spheres.
4. At what energy (in units of  $kT$ ) is the Fermi function within 1 % of the Maxwell-Boltzmann distribution function? What is the corresponding probability of occupancy?
5. Calculate the Fermi function at 6.5 eV if  $E_F = 6.25$  eV and  $T = 300$  K. Repeat at  $T = 950$  K assuming that the Fermi energy does not change. At what temperature does the probability that an energy level at  $E = 5.95$  eV is empty equal 1 %.
6. Calculate the effective density of states for electrons and holes in germanium, silicon and gallium arsenide at room temperature and at 100 °C. Use the effective masses for density of states calculations.
7. Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at room temperature (300 K). Repeat at 100 °C. Assume that the energy bandgap is independent of temperature and use the room temperature values.
8. Calculate the position of the intrinsic energy level relative to the midgap energy

$$E_{midgap} = (E_c + E_v)/2$$

in germanium, silicon and gallium arsenide at 300 K. Repeat at  $T = 100$  °C.

9. Calculate the electron and hole density in germanium, silicon and gallium arsenide if the Fermi energy is 0.3 eV above the intrinsic energy level. Repeat if the Fermi energy is 0.3 eV below the conduction band edge. Assume that  $T = 300$  K.
10. The equations (2.6.37) and (2.6.38) derived in section 2.6 are only valid for non-degenerate semiconductors (i.e.  $E_v + 3kT < E_F < E_c - 3kT$ ). Where exactly in the derivation was the assumption made that the semiconductor is non-degenerate?
11. A silicon wafer contains  $10^{16}$  cm<sup>-3</sup> electrons. Calculate the hole density and the position of the intrinsic energy and the Fermi energy at 300 K. Draw the corresponding band diagram to scale, indicating the conduction and valence band edge, the intrinsic energy level and the Fermi energy level. Use  $n_i = 10^{10}$  cm<sup>-3</sup>.
12. A silicon wafer is doped with  $10^{13}$  cm<sup>-3</sup> shallow donors and  $9 \times 10^{12}$  cm<sup>-3</sup> shallow acceptors. Calculate the electron and hole density at 300 K. Use  $n_i = 10^{10}$  cm<sup>-3</sup>.
13. The resistivity of a silicon wafer at room temperature is 5 Ω-cm. What is the doping density? Find all possible solutions.
14. How many phosphorus atoms must be added to decrease the resistivity of  $n$ -type silicon at room temperature from 1 Ω-cm to 0.1 Ω-cm. Make sure you include the doping dependence

of the mobility. State your assumptions.

15. A piece of  $n$ -type silicon ( $N_d = 10^{17} \text{ cm}^{-3}$ ) is uniformly illuminated with green light ( $\lambda = 550 \text{ nm}$ ) so that the optical power density in the material equals  $1 \text{ mW/cm}^2$ . a) Calculate the generation rate of electron-hole pairs using an absorption coefficient of  $10^4 \text{ cm}^{-1}$ . b) Calculate the excess electron and hole density using the generation rate obtained in (a) and a minority carrier lifetime due to Shockley-Read-Hall recombination of  $0.1 \text{ ms}$ . c) Calculate the electron and hole quasi-Fermi energies (relative to  $E_i$ ) based on the excess densities obtained in (b).
16. A piece of intrinsic silicon is instantaneously heated from  $0 \text{ K}$  to room temperature ( $300 \text{ K}$ ). The minority carrier lifetime due to Shockley-Read-Hall recombination in the material is  $1 \text{ ms}$ . Calculate the generation rate of electron-hole pairs immediately after reaching room temperature. ( $E_t = E_i$ ). If the generation rate is constant, how long does it take to reach thermal equilibrium?
17. Calculate the conductivity and resistivity of intrinsic silicon. Use  $n_i = 10^{10} \text{ cm}^{-3}$ ,  $\mu_n = 1400 \text{ cm}^2/\text{V-sec}$  and  $\mu_p = 450 \text{ cm}^2/\text{V-sec}$ .
18. Consider the problem of finding the doping density, which provides the maximum possible resistivity of silicon at room temperature. ( $n_i = 10^{10} \text{ cm}^{-3}$ ,  $\mu_n = 1400 \text{ cm}^2/\text{V-sec}$  and  $\mu_p = 450 \text{ cm}^2/\text{V-sec}$ .)

Should the silicon be doped at all or do you expect the maximum resistivity when dopants are added?

If the silicon should be doped, should it be doped with acceptors or donors (assume that all dopants are shallow).

Calculate the maximum resistivity, the corresponding electron and hole density and the doping density.

19. The electron density in a piece of silicon ( $T = 300\text{K}$ ) is twice the intrinsic density. Calculate the corresponding hole density, the donor density and the Fermi energy relative to the intrinsic energy. Repeat for  $n = 5 n_i$  and  $n = 10 n_i$ . Also repeat for  $p = 2 n_i$ ,  $p = 5 n_i$  and  $p = 10 n_i$ , calculating the electron and acceptor density as well as the Fermi energy relative to the intrinsic energy level.
20. The expression for the Bohr radius can also be applied to the hydrogen-like atom consisting of an ionized donor and the electron provided by the donor. Modify the expression for the Bohr radius so that it applies to this hydrogen-like atom. Calculate the Bohr radius of an electron orbiting around the ionized donor in silicon. ( $\epsilon_r = 11.9$  and  $m_e^* = 0.26 m_0$ )
21. Calculate the density of electrons per unit energy (in electron volt) and per unit area (per cubic centimeter) at  $1 \text{ eV}$  above the band minimum. Assume that  $m_e^* = 1.08 m_0$ .
22. Calculate the probability that an electron occupies an energy level, which is  $3kT$  below the Fermi energy. Repeat for an energy level, which is  $3kT$  above the Fermi energy.
23. Calculate and plot as a function of energy the product of the probability that an energy level

is occupied with the probability that that same energy level is not occupied. Assume that the Fermi energy is zero and that  $kT = 1$  eV

24. The effective mass of electrons in silicon is  $0.26 m_0$  and the effective mass of holes is  $0.36 m_0$ . If the scattering time is the same for both carrier types, what is the ratio of the electron mobility and the hole mobility.
25. Electrons in silicon carbide have a mobility of  $1000 \text{ cm}^2/\text{V}\cdot\text{sec}$ . At what value of the electric field do the electrons reach a velocity of  $3 \times 10^7 \text{ cm/s}$ ? Assume that the mobility is constant and independent of the electric field. What voltage is required to obtain this field in a 5-micron thick region? How much time do the electrons need to cross the 5 micron thick region?
26. A piece of silicon has a resistivity which is specified by the manufacturer to be between 2 and 5 Ohm cm. Assuming that the mobility of electrons is  $1400 \text{ cm}^2/\text{V}\cdot\text{sec}$  and that of holes is  $450 \text{ cm}^2/\text{V}\cdot\text{sec}$ , what is the minimum possible carrier density and what is the corresponding carrier type? Repeat for the maximum possible carrier density.
27. A silicon wafer has a 2-inch diameter and contains  $10^{14} \text{ cm}^{-3}$  electrons with a mobility of  $1400 \text{ cm}^2/\text{V}\cdot\text{sec}$ . How thick is the wafer if the resistance between the front and back surface equals 0.1 Ohm?
28. The electron mobility in germanium is  $1000 \text{ cm}^2/\text{V}\cdot\text{sec}$ . If this mobility is due to impurity and lattice scattering and the mobility due to lattice scattering only is  $1900 \text{ cm}^2/\text{V}\cdot\text{sec}$ , what is the mobility due to impurity scattering only?
29. An atomic system consists of three energy levels with energy 0, 10 and 20 meV, which can contain a maximum of 1000, 2000 and 1000 electrons, respectively. The total energy of the system in thermal equilibrium is 25 eV and the total number of electrons is 2000. Calculate the Fermi energy and the temperature. (Challenge problem)
30. A  $20 \text{ }\mu\text{m}$  thin piece of gallium arsenide consists of two regions with a different carrier life time, namely  $\tau = 20 \text{ ns}$  for  $0 \leq x \leq 10 \text{ }\mu\text{m}$  and  $\tau = \infty$  for  $10 \leq x \leq 20 \text{ }\mu\text{m}$ . The material is illuminated with light so that  $10^{22} \text{ cm}^{-2}\text{s}^{-1}$  electron-hole pairs are created at  $x = 20 \text{ }\mu\text{m}$ . Calculate the steady state electron and hole density at  $x = 0, 10$  and  $20 \text{ }\mu\text{m}$ . (Assume  $\mu_n = \mu_p = 1000 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $T = 300 \text{ K}$ )
31. A 10-micron thin piece of  $n$ -type silicon ( $N_d = 10^{17} \text{ cm}^{-3}$ ) is uniformly illuminated with light, resulting in an electron-hole pair generation rate of  $10^{23} \text{ cm}^{-3}\text{s}^{-1}$ . The silicon is contacted only on one side with an ideal Ohmic contact. Assume there is no recombination and no electric field in the semiconductor. (Use  $\mu_n = 1000 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $\mu_p = 300 \text{ cm}^2/\text{V}\cdot\text{s}$ )
  - a) What is the total current measured at the contact. Justify your answer.
  - b) What properties of the minority carrier density can you identify without actually solving the problem.
  - c) How would you solve for the minority carrier density.

- d) Calculate the minority carrier density throughout the silicon.
- e) Calculate the hole current density at the Ohmic contact.

32. Consider a semiconductor with two parabolic conduction bands having conduction band minima  $E_{c1}$  and  $E_{c2}$  with effective masses  $m_{e1} = 0.06 m_0$  and  $m_{e2} = 0.4 m_0$  and mobility  $\mu_{n1} = 8000 \text{ cm}^2/\text{V-s}$  and  $\mu_{n2} = 1000 \text{ cm}^2/\text{V-s}$ . The conduction band minimum  $E_{c2}$  is 30 meV higher than  $E_{c1}$ .

- a) Find an expression as well as the numeric value for the effective density of states,  $N_c^*$ , taking into account both conduction bands, so that the usual expressions for the intrinsic carrier concentration and the intrinsic Fermi energy still hold, namely:

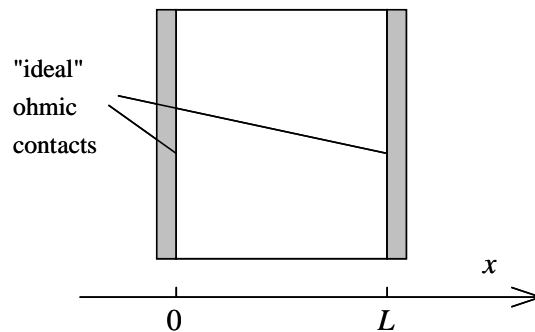
$$n_i = \sqrt{N_c^* N_v} \exp\left(-\frac{E_g}{kT}\right)$$

$$E_i = \frac{E_{c1} + E_v}{2} - \frac{kT}{2} \ln \frac{N_c^*}{N_v}$$

Where the energy bandgap,  $E_g$ , equals  $E_{c1} - E_v$  and  $E_v$  is the top of the valence band.

- b) Find an expression as well as the numeric value for the effective electron mobility  $\mu_n^*$ , again taking into account both conduction bands so that the usual expression for the conductivity still holds, namely  $\sigma_n = q n \mu_n^*$  where  $n$  is the combined density of electrons in both conduction bands.

33. An intrinsic piece of GaAs ( $n_i = 2 \times 10^6 \text{ cm}^{-3}$ ) of length  $L (= 1 \text{ } \mu\text{m})$  is uniformly illuminated with light yielding an electron-hole pair generation rate of  $10^{22} \text{ cm}^{-3} \text{ s}^{-1}$ . On both ends (at  $x = 0$  and  $x = L$ ) the material is contacted with “ideal” ohmic contacts. Calculate the maximum value of the steady-state excess hole density in the material under illumination. The hole diffusion length,  $L_p$ , equals  $\frac{L}{\ln(2)}$  and the hole mobility is  $500 \text{ cm}^2/\text{V-s}$ . Assume that there is no electric field in the semiconductor and  $T = 300 \text{ K}$ .



34. An intrinsic piece of GaAs ( $n_i = 2 \times 10^6 \text{ cm}^{-3}$ ) of length  $L (= 1 \text{ } \mu\text{m})$  is uniformly illuminated with light yielding a electron-hole pair generation rate of  $10^{22} \text{ cm}^{-3} \text{ s}^{-1}$ . On one end (at  $x =$

0) the material is contacted with an “ideal” ohmic contact, while on the other end (at  $x = L$ ) the current is limited by a finite recombination velocity,  $s$ , ( $s = 10^5$  cm/s), as described by the following relation between the excess hole density,  $\delta p$ , and the hole current density,  $J_p$ :

$$J_p(x=L) = q \delta p(x=L) s$$

Calculate the maximum value of the steady-state excess hole density in the material under illumination. The hole diffusion length,  $L_p$ , equals  $\frac{L}{\ln(2)}$  and the hole mobility is  $500 \text{ cm}^2/\text{V}\cdot\text{s}$ . Assume that there is no electric field in the semiconductor and  $T = 300 \text{ K}$ .

