3.3.6. Derivation of Schottky barrier lowering

We now derive equation (3.3.12) by calculating the potential due to the image charge and adding it to the potential within the depletion region.

The electrostatic force between the two particles, one with a positive electronic charge and the other with a negative electronic charge, which are as shown in Figure 3.3.3, both a distance, x, away from the interface ($x = 0$), is given by:

$$F(x) = q\mathcal{E}_I(x) = \frac{-q^2}{4\pi\varepsilon_s(2x)^2} \tag{3.3.13}$$

The corresponding potential, $\phi_{image}(x)$, equals:

$$\phi_{image}(x) = -\int_x^\infty \mathcal{E}_{image}(x)dx = \frac{q}{16\pi\varepsilon_s x} \tag{3.3.14}$$

which combined with the potential variation due to the electric field yields the following potential energy, $V(x)$, versus position, x:

$$V(x) = -q\mathcal{E}_{\max}x - \frac{q^2}{16\pi\varepsilon_s x} \tag{3.3.15}$$

where the field due to the charge in the depletion region is assumed to be constant and set equal to the maximum field, $\mathcal{E}_{\max}$:

$$\mathcal{E}_{\max} = \sqrt{\frac{qN_d\,2(\phi_i - V_a)}{\varepsilon_s}} \tag{3.3.16}$$

At this point the question arises why a single electron can noticeably alter the potential, while the depletion layer contains significantly more charge. To understand this, one has to realize that we have assumed that the charge in the depletion region is not quantized, but instead is distributed throughout the depletion layer. This assumption does provide the correct average potential, but it does not accurately reflect the local potential variations due to the individual charges of the ionized donors or mobile electrons. As we assumed that the single electron is far away from all other charges in the semiconductor, the potential energy due to all those charges is close to the average potential energy.

The potential energy, $V(x)$, due to the distributed charge of the ionized donors and a single electron reaches its maximum value at:

$$x_{\max} = \sqrt{\frac{q}{16\pi\varepsilon_s \mathcal{E}_{\max}}} \tag{3.3.17}$$

and the corresponding maximum value of the potential energy is:

$$V_{\max} = -q\Delta\phi_B \tag{3.3.18}$$

where $\Delta\phi_B$ is the barrier height reduction given by:

$$\Delta\phi_B = \sqrt{\frac{q\mathcal{E}_{\max}}{4\pi\varepsilon_s}} \tag{3.3.19}$$