3.3.7. Solution to Poisson's Equation

To assess the error made when using the full depletion approximation we now derive the correct solution by solving Poisson's equation analytically[2]. The actual solution for the potential is then obtained by numerically integrating the expression for the electric field. We start from the charge density, r, in a semiconductor for the general case where electrons, holes, ionized acceptors and ionized donors are present:

$$r(f) = q(p + N_d^+ - n - N_a^-) \tag{3.3.19}$$

Where f is the potential in the semiconductor. The potential is chosen to equal zero deep into the semiconductor. For an n-type semiconductor without acceptors or free holes this can be further reduced to:

$$r(f) = qN_d(1 - \exp(\frac{qf}{kT})) \tag{3.3.20}$$

assuming the semiconductor to be non-degenerate and fully ionized. A similar expression can be obtained for p-type material. Poisson's law can then be rewritten as:

$$\frac{d^2f}{dx^2} = -\frac{r(f)}{e_s} = -qN_d(1 - \exp(\frac{qf}{kT})) \tag{3.3.21}$$

Multiplying both sides with df/dx, this equation can be integrated between an arbitrary point x and infinity. The electric field at infinity (deep in the semiconductor) is taken to be zero. The electric field for a given potential is then:

$$E(f) = \sqrt{2\int_0^f \frac{-qN_d}{e_s}(1 - \exp(\frac{qf}{kT}))df} = sign(f)\frac{V_t}{L_D}\sqrt{2[\exp(\frac{f}{V_t}) - (\frac{f}{V_t}) - 1]} \tag{3.3.22}$$

Where the sign function equals +1 or -1 depending on the sign of f and L_D is the Debye length given by, $L_D = \sqrt{\frac{e_s kT}{q^2 N_d}}$. Equation (3.3.22) is plotted in Figure 3.3.4 using normalized parameters. Depletion occurs for negative potentials while accumulation occurs for positive potentials.

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[2]This derivation follows that of Goodman and Perkins, J. Appl. Phys. **35**, p 3351, 1964.

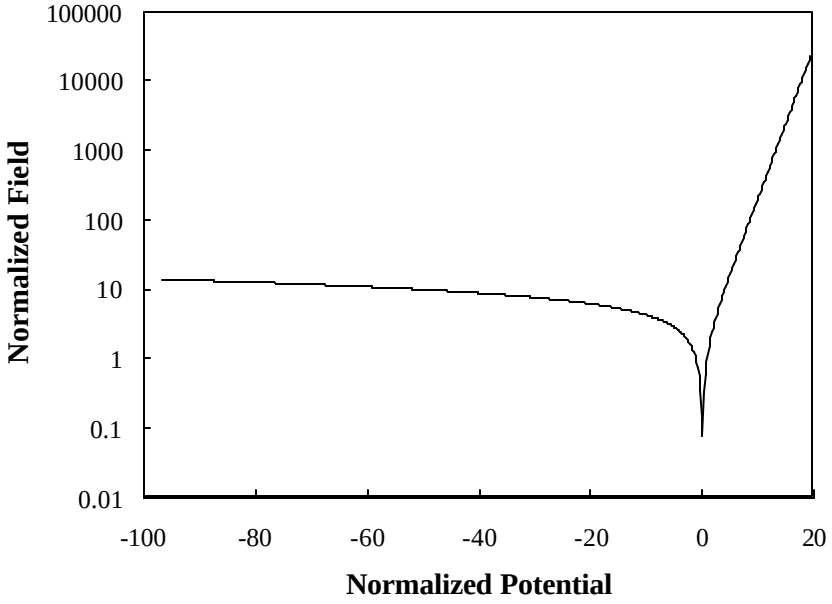

Figure 3.3.4 Absolute value of the normalized electric field, $|E|\ L_D/V_t$, versus normalized potential, f/V_t

Applying Gauss's law ($Q = e_s E$) we the find the relation between the total charge in the semiconductor region and the total potential across the semiconductor. The capacitance can also be obtained from:

$$C = \left|\frac{dQ}{dV_a}\right| = e_s \left|\frac{dE}{dV_a}\right| = \frac{e_s}{L_D}\left|\frac{\exp(\frac{f}{V_t}) - 1}{\sqrt{2[\exp(\frac{f}{V_t}) - (\frac{f}{V_t}) - 1]}}\right| \tag{3.3.23}$$

where f_s is the potential across the semiconductor and equals $-f_i + V_a$. This expression can be approximated for $f_s < 0$ and $|f_s| >> V_t$ yielding:

$$C = \frac{e_s}{L_D}\frac{1}{\sqrt{2\frac{f_i - V_a - V_t}{V_t}}} = \sqrt{\frac{e_s q N_d}{2(f_i - V_a - V_t)}} \tag{3.3.24}$$

This expression equals (3.3.10) as derived using the full depletion approximation, except for the added term, V_t, in the denominator. This expression yields the capacitance value with a relative accuracy better than 0.3 % for $V_a < f_i - 6V_t$.

3.3.7.1.Numeric solution

A numeric solution can be obtained by integrating equation (3.3.21). The solution to the energy band diagram, the charge density, the electric field and the potential are shown in the figures below: Integration was started four Debye lengths to the right of the edge of the depletion region as obtained using the full depletion approximation. Initial conditions were obtained by assuming the potential at the starting point to be adequately expressed by a solution to the homogenous equation:

$$f = V_t \exp\left(-\frac{(x - x_d)}{L_D}\right) \tag{3.3.25}$$

Shown are solutions for a gold-silicon M-S junction with Φ_M = 4.75V, c = 4.05V, N_d = 10^{16} cm^{-3} and e_s/e_0 = 11.9.

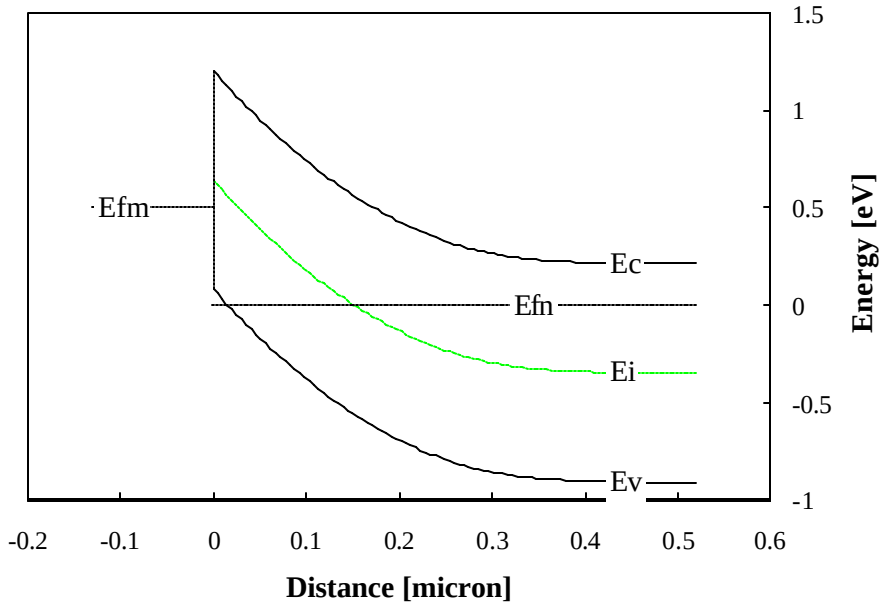

Figure 3.3.5 Energy band diagram of an M-S junction

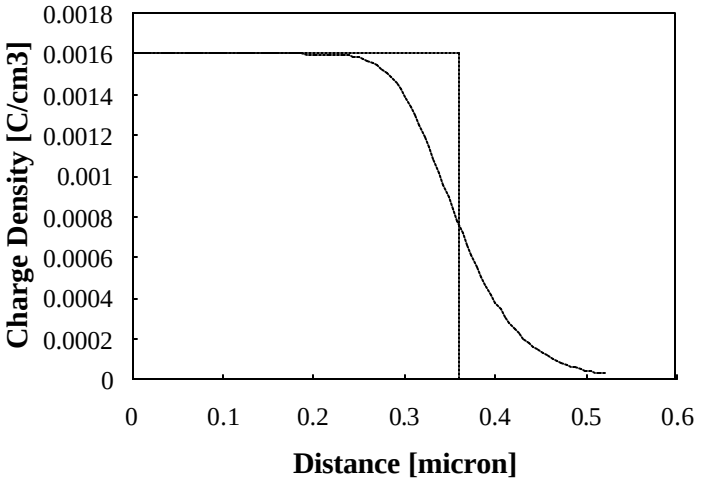

Figure 3.3.6 Charge density versus position in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

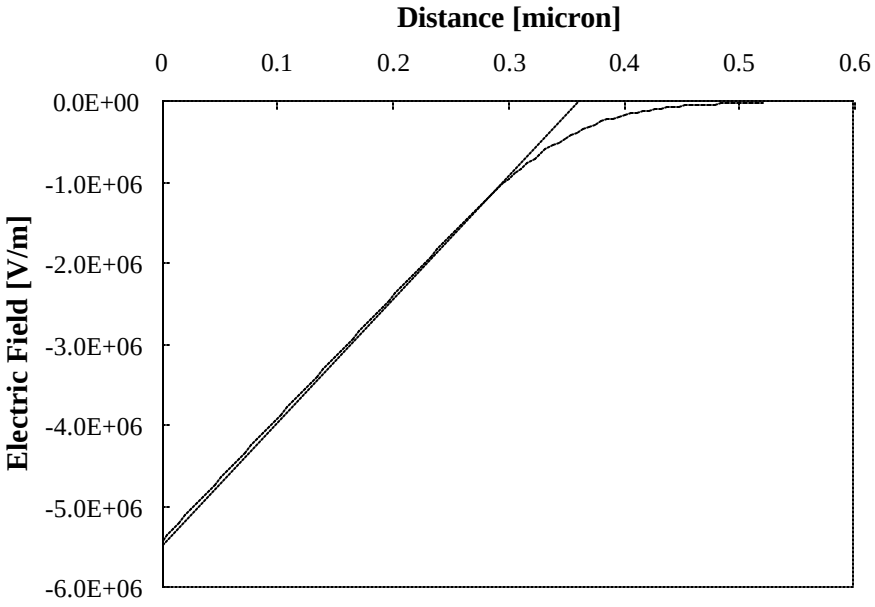

Figure 3.3.7 Electric field versus distance in a M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

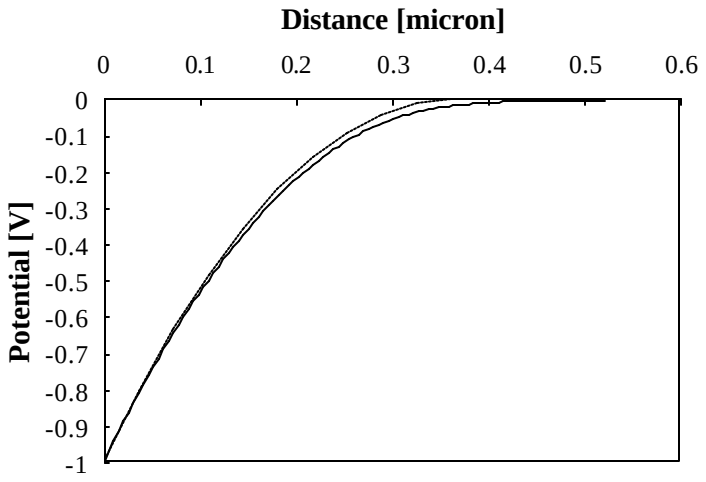

Figure 3.3.8 Potential versus distance of an M-S junction. The solid line is the numeric solution, and the dotted line is the solution based on the full depletion approximation.

3.3.7.2. Depletion at the Metal-Semiconductor interface

Most metal semiconductor contacts have a depletion region adjacent to the interface. We distinguish between the case where a large potential variation is found across the semiconductor, for which only a small correction is obtained compared to the full depletion approximation, and the case where a small potential variation exists across the semiconductor, for which the full depletion approximation does not apply.

3.3.7.2.1. Large potential approximation

If the potential difference across the semiconductor is larger than the thermal voltage, or $f_s = V_a - f_i < 0$ and $|V_a - f_i| >> kT/q$ we find the effective depletion layer width, x_d, defined as the ratio of the total depletion layer charge to the charge density of the fully ionized donors, to be:

$$x_d = \frac{Q_d}{qN_d} = L_D\sqrt{2\left(\frac{f_i - V_a - V_t}{V_t}\right)} \tag{3.3.26}$$

where L_D is the extrinsic Debye length of the semiconductor, which is given by:

$$L_D = \sqrt{\frac{e_s kT}{q^2 N_d}} \tag{3.3.27}$$

The small signal capacitance can be expressed by:

$$C = \frac{dQ_d}{dV_a} = \frac{e_s}{L_D}\sqrt{\frac{V_t}{2(f_i - V_a - V_t)}} = \frac{e_s}{x_d} \tag{3.3.28}$$

where Q_d is the total charge per unit area in the depletion layer. This result differs from the one obtained by using the full depletion approximation in that the applied voltage is increased by the thermal voltage. However the capacitance is still the ratio of the dielectric constant to the depletion layer width.

3.3.7.2.2. Small potential approximation

If the potential difference across the semiconductor is smaller than the thermal voltage, or $f = V_a - f_i < 0$ and $|V_a - f_i| < kT/q$, the depletion layer width is proportional to the Debye length and the applied voltage:

$$x_d = L_D \frac{f_i - V_a}{V_t} \tag{3.3.29}$$

and the capacitance is constant, independent of the applied voltage:

$$C = \frac{e_s}{L_D} \tag{3.3.30}$$

3.3.7.3.Accumulation at the Metal-Semiconductor interface

Accumulation occurs at the semiconductor metal interface if the Fermi energy of the metal lies between the conduction band edge and the Fermi energy in the n-type semiconductor, or $\Phi_S > \Phi_M > c$. A similar condition can be defined for p-type material. Equation [3.1.10] applies for depletion as well as accumulation. However it does not provide a solution for the electric field and potential as a function of position. Instead we start again from the integral formulation of equation [3.1.10] but set the potential equal to zero at the interface and integrate from 0 to x. We also assume that the electron concentration at the surface, n_S is much larger than the donor concentration. Using this convention, equation [3.1.10] can be rewritten as:

$$E(f) = \sqrt{\frac{2n_s kT}{e_s}}\exp(\frac{qf}{2kT}) = -\frac{df}{dx} \tag{3.3.31}$$

integrating this equation again from 0 to x yields:

$$\sqrt{\frac{2qn_s kT}{e_s}}x = 2V_t[\exp(-\frac{f}{2V_t}) - 1] \tag{3.3.32}$$

from which the charge density can be obtained:

$$r(x) = -qn_s \exp(\frac{f}{V_t}) = \frac{-qn_s}{(1+\frac{x}{\sqrt{2}L_D})^2} \tag{3.3.33}$$

Integration of the charge density yields the electric field.

$$E(x) = \frac{\sqrt{2}kT}{L_D q}\frac{1}{1+\frac{x}{\sqrt{2}L_D}} \tag{3.3.34}$$

The width of the accumulation layer is obtained by solving the expression for the potential for x with $f(x_d) = f_i - V_a$.

$$x_d = \sqrt{2}L_D[\exp(\frac{|f_i - V_a|}{2V_t}) - 1] \tag{3.3.35}$$

The correct solution can also be obtained by integrating [3.1.10]. A solution for a M-S junction with $\Phi_M = 4.2$V, $c = 4.05$V, $N_d = 10^{16}$ cm^{-3} and $e_s/e_0 = 11.9$ is shown in the figures below.

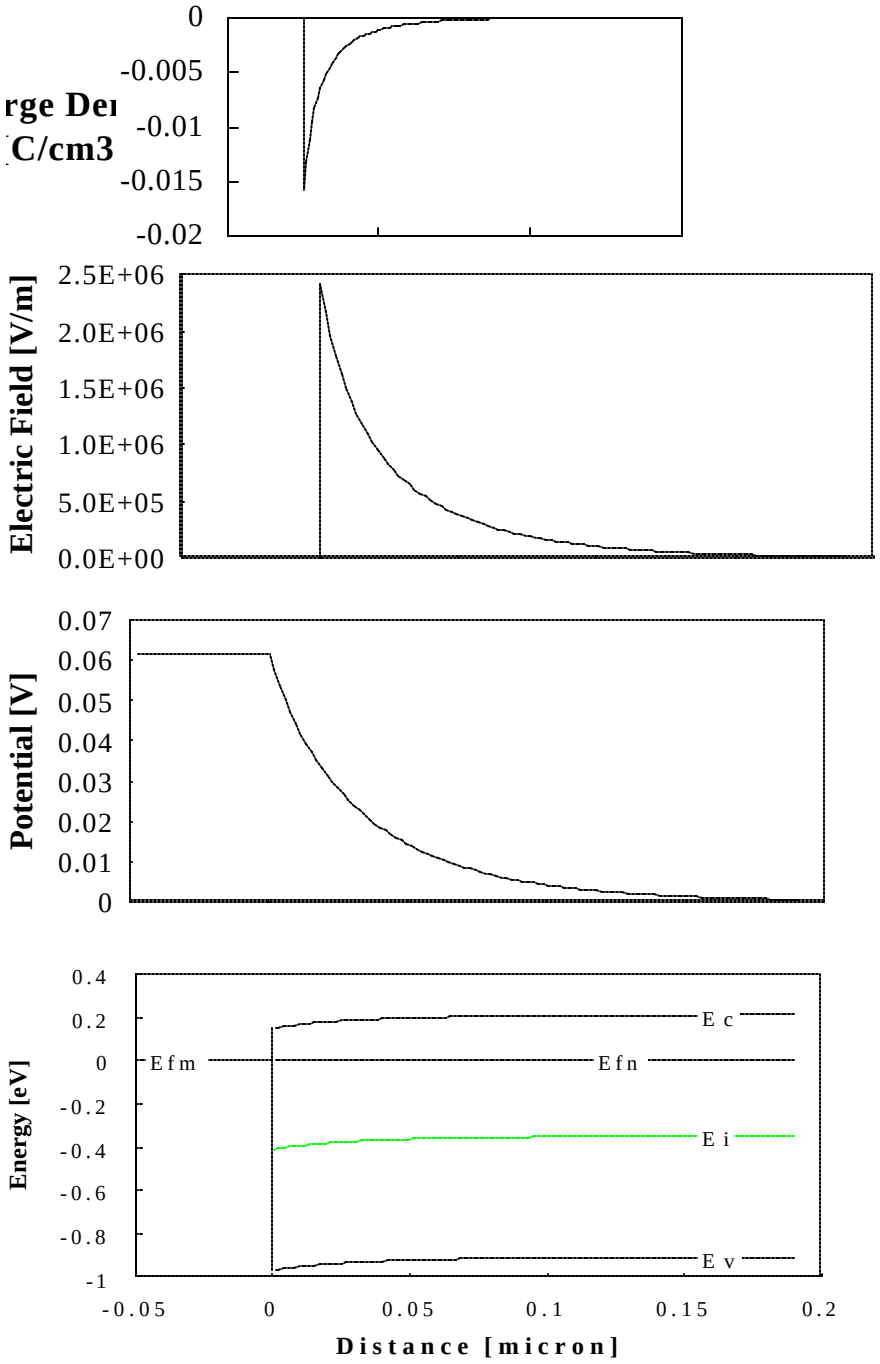

Figure 3.3.9 Charge density, electric field, potential and energy band diagram of a metal-semiconductor junction