4.10. Laser diodes

4.10.1. Emission, Absorption and modal gain

The analysis of a semiconductor laser diode requires a detailed knowledge of the modal gain, which quantifies the amplification of light confined to the lasing mode. To find the modal gain, one starts from the notion that the emission as well as absorption of photons, requires the conservation of energy and momentum of all particles involved in the process. The conservation of energy requires that the photon energy equals the difference between the electron and hole energy:

$$E_{ph} = E_n - E_p \tag{4.10.1}$$

with

$$E_n = E_c + E_{1n} + \frac{\hbar^2 k_n^2}{2m_n^*} \tag{4.10.2}$$

$$E_p = E_v - E_{1p} + \frac{\hbar^2 k_p^2}{2m_p^*} \tag{4.10.3}$$

The conservation of momentum requires that the electron momentum equals that of the empty state it occupies in the valence band plus the momentum of the photon:

$$k_n = k_p + k_{ph} \tag{4.10.4}$$

The photon momentum is much smaller than that of the electron and hole, so that the electron and hole momentum are approximately equal. As a result we can replace k_n and k_p by a single variable k. Equations (4.10.1), (4.10.2), (4.10.3) and (4.10.4) then result in:

$$E_{ph} = E_{g,qw1} + \frac{\hbar^2 k^2}{2m_r^*} \tag{4.10.5}$$

where $E_{g,qw1}$ is the energy between the lowest electron energy in the conduction band and the lowest hole energy in the valence band. m_r^* is the reduced effective mass given by:

$$\frac{1}{m_r^*} = \frac{1}{m_n^*} + \frac{1}{m_p^*} \tag{4.10.6}$$

The electron and hole energies, E_n and E_p, can then be expressed as a function of the photon energy by:

$$E_n = E_c + E_{1n} + (E_{ph} - E_{g,qw1})\frac{m_r^*}{m_n^*} \tag{4.10.7}$$

$$E_p = E_v - E_{1p} - (E_{ph} - E_{g,qw1})\frac{m_r^*}{m_p^*} \qquad (4.10.8)$$

The emission and absorption spectra ($b(E_{ph})$ and $a(E_{ph})$) of a quantum well depend on the density of states and the occupancy of the relevant states in the conduction and valence band. Since the density of states in the conduction and valence band are constant in a quantum well, the emission and absorption can be expressed as a product of a maximum emission and absorption rate and the probability of occupancy of the conduction and valence band states, namely:

$$b(E_{ph}) = b_{\max} f_n(E_n)[1 - f_p(E_p)] \qquad (4.10.9)$$

$$a(E_{ph}) = a_{\max}[1 - f_n(E_n)] f_p(E_p) \qquad (4.10.10)$$

Stimulated emission occurs if an incoming photon triggers the emission of another photon. The net gain in the semiconductor is the stimulated emission minus the absorption. The maximum stimulated emission equals the maximum absorption since the initial and final states are simply reversed so that the transition rates as calculated based on the matrix elements are the same. The net gain is then given by:

$$g(E_{ph}) = b(E_{ph}) - a(E_{ph}) = g_{\max}[f_n(E_n) - f_p(E_p)] \qquad (4.10.11)$$

where the maximum stimulated emission and the maximum absorption were replaced by the maximum gain, g_{max}. The normalized gain spectrum is shown in Figure 4.10.1 for different values of the carrier density. The two staircase curves indicate the maximum possible gain and the maximum possible absorption in the quantum well.

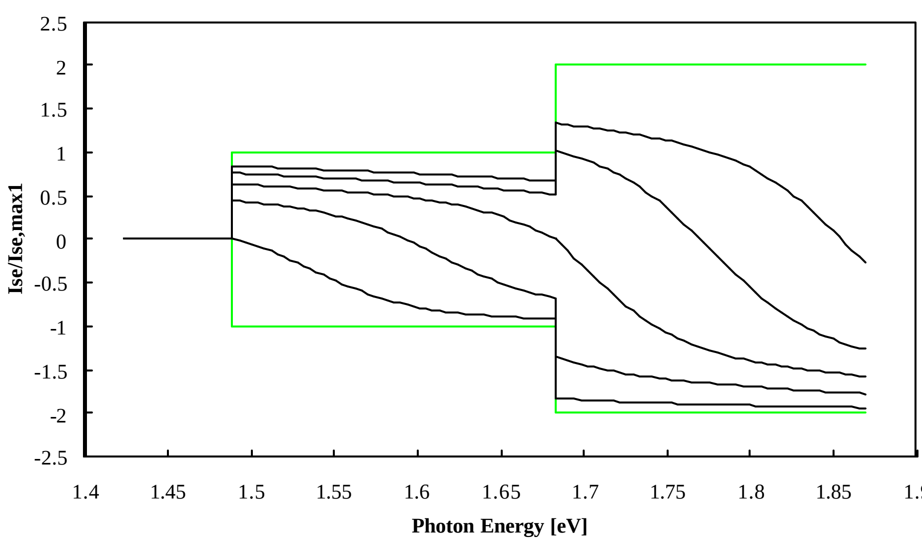

Figure 4.10.1 Normalized gain versus photon energy of a 10nm GaAs quantum well for a carrier density of 10^{12} (lower curve), 3 x 10^{12}, 5 x 10^{12}, 7 x 10^{12} and 9 x 10^{12}

(upper curve) cm^{-2}.

The theoretical gain curve of Figure 4.10.1 exhibits a sharp discontinuity at $E_{ph} = E_{g,qw1}$. The gain can also be expressed as a function of the carrier densities, N and P, when assuming that only one electron and one hole level is occupied:

$$g(E_{ph}) = g_{\max}\left[\frac{1-\exp(-\frac{N}{N_{c,qw}})}{1+\exp(-\frac{N}{N_{c,qw}})\left[\exp(\frac{E_{ph}-E_{g,qw1}}{kT}\frac{m_r^*}{m_n^*})-1\right]} - \frac{\exp(-\frac{P}{N_{v,qw}})}{(1-\exp(-\frac{P}{N_{v,qw}}))\exp(\frac{E_{g,qw1}-E_{ph}}{kT}\frac{m_r^*}{m_p^*})+\exp(-\frac{P}{N_{v,qw}})}\right] \quad (4.10.12)$$

The peak value at $E_{ph} = E_{g,qw1}$, assuming quasi-neutrality ($N = P$) is then:

$$g_{peak} = g(E_{g,qw1}) = g_{\max}\left(1-\mathrm{e}^{-N/N_c} - e^{-N/N_v}\right) \quad (4.10.13)$$

The maximum gain can be obtained from the absorption of light in bulk material since the wavefunction of a free electron in bulk material is the same as the wavefunction in an infinite stack of infinitely deep quantum wells, provided the barriers are infinitely thin and placed at the nodes of the bulk wavefunction. This means that for such a set of quantum wells the absorption would be the same as in bulk provided that the density of states is also the same. This is the case for $E_{ph} = E_{qw1}$ so that the maximum gain per unit length is given by:

$$g_{\max} = K\sqrt{E_{qw1}-E_1} = \frac{K}{2}\sqrt{\frac{h^2}{2m_r^*}}\frac{1}{L_x} \quad (4.10.14)$$

where L_x is the width of the quantum well. This expression shows that the total gain of a single quantum well due to a single quantized level is independent of the width[1]. The corresponding value for GaAs quantum wells is 0.006 or 0.6%.

Experimental gain curves do not show the discontinuity at $E_{ph} = E_{qw1}$ due to inter-carrier scattering which limits the lifetime of carriers in a specific state. The line width of a single set of electron and hole levels widens as a function of the scattering time, which disturbs the phase of the atomic oscillator. Therefore, an approximation to the actual gain curve can be obtained by convoluting (6.2.10) with a Lorenzian line shape function:

---

[1] There is a weak dependence of m^* on the width of the well.

$$g(E_{ph}) = \int g_{\max}[F_n(E_n) - F_p(E_p)] \frac{\Delta n}{2p[(n - \frac{E_{ph}}{h})^2 + (\frac{\Delta n}{2})^2]} dn \tag{4.10.15}$$

with $\Delta n = \frac{1}{p}\frac{1}{t}$, where t is the carrier collision time in the quantum well. The original and convoluted gain curves are shown in **Figure 4.10.2**.

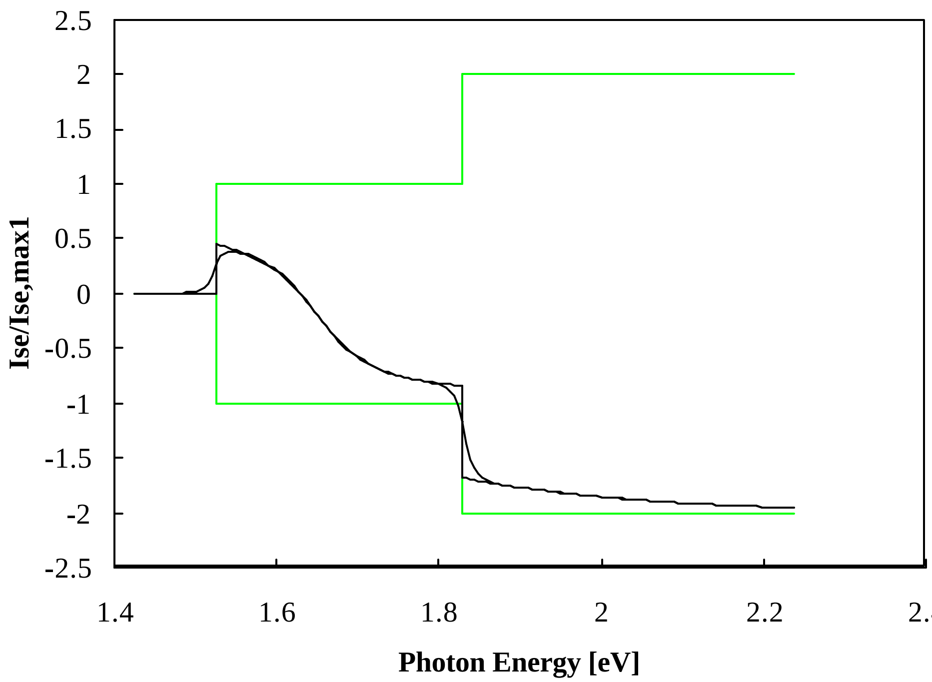

Figure 4.10.2 Original and convoluted gain spectrum of a 10 nm GaAs quantum well with a carrier density of 3 x 10^{12} cm^{-2} and a collision time of 0.09 ps.

For lasers with long cavities such as edge-emitter lasers, one finds that the longitudinal modes are closely spaced so that lasing will occur at or close to the peak of the gain spectrum. It is therefore of interest to find an expression for the peak gain as a function of the carrier density[2]. A numeric solution is shown in **Figure 4.10.3** where the peak gain is normalized to the maximum value of the first quantized energy level. Initially, the gain peak is linear with carrier concentration but saturates because of the constant density of states, until the gain peak associated with the second quantized level takes over. Since the peak gain will be relevant for lasing we will consider it more closely. As a first order approximation we will set the peak gain

[2]Experimental values for the gain versus current density can be found in: G. Hunziker, W. Knop and C. Harder, "Gain Measurements on One, Two and Three Strained GaInP Quantum Well Laser Diodes", IEEE Trans. Quantum Electr., Vol. 30, p 2235-2238, 1994.

$g(N)$ equal to:

$$g(N) = l(N - N_{tr}) \tag{4.10.16}$$

where l is the differential gain coefficient. This approximation is only valid close to $N = N_{tr}$, and even more so for quantum well lasers as opposed to double-hetero-structure lasers. An approximate value for the differential gain coefficient of a quantum well can be calculated from (6.2.13) yielding:

$$l = g_{\max}\left[\frac{e^{-N_{tr}/N_c}}{N_c} + \frac{e^{-N_{tr}/N_v}}{N_v}\right] \tag{4.10.17}$$

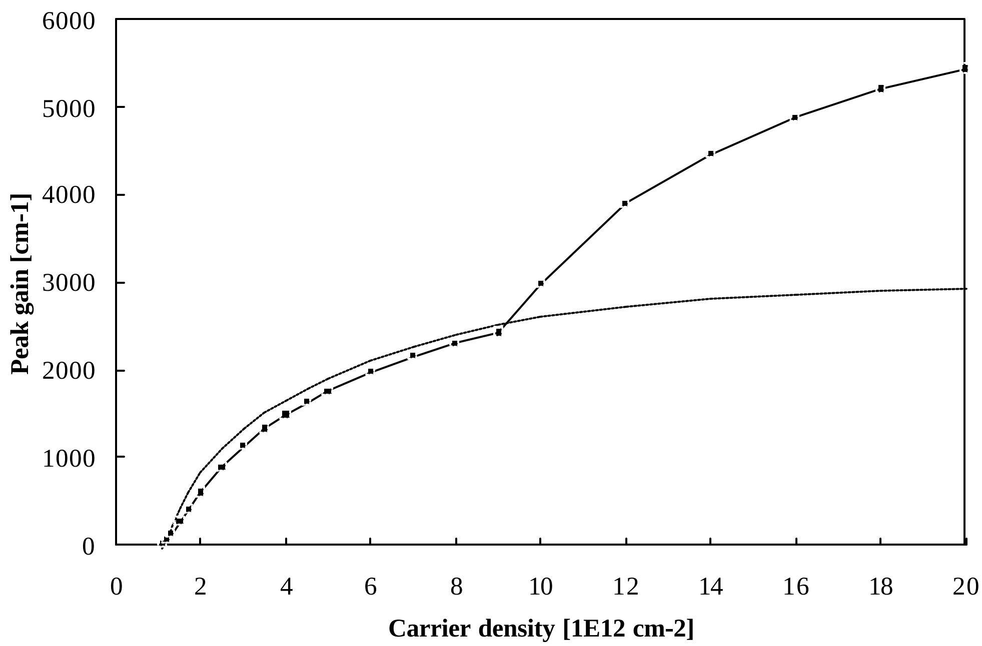

Figure 4.10.3 Calculated gain versus carrier density for a 10 nm GaAs quantum well (solid line) compared to equation [6.2.13]

From **Figure 4.10.3** one finds that the material becomes "transparent" when the gain equals zero or:

$$g(N_{tr}) = 0 = g_{\max}[F_n(E_n)(1 - F_p(E_p)) - (1 - F_n(E_n))F_p(E_p)] \tag{4.10.18}$$

which can be solved yielding:

$$E_{ph} = E_n - E_p = E_{Fn} - E_{Fp} = qV_a \tag{4.10.19}$$

The transparency current density is defined as the minimal current density for which the material becomes transparent for any photon energy larger than or equal to $E_{g,qw1}$. This means that the transparency condition is fulfilled for $V_a = \frac{E_{g.qw1}}{q}$. The corresponding carrier density is referred to as N_{tr}, the transparency carrier density. The transparency carrier density can be obtained from by setting g_{max} = 0, yielding

$$N_{tr} = -N_c \ln(1 - e^{N_{tr}/N_v}) \tag{4.10.20}$$

This expression can be solved by iteration for $N_v > N_c$. The solution is shown in Figure 4.10.4.

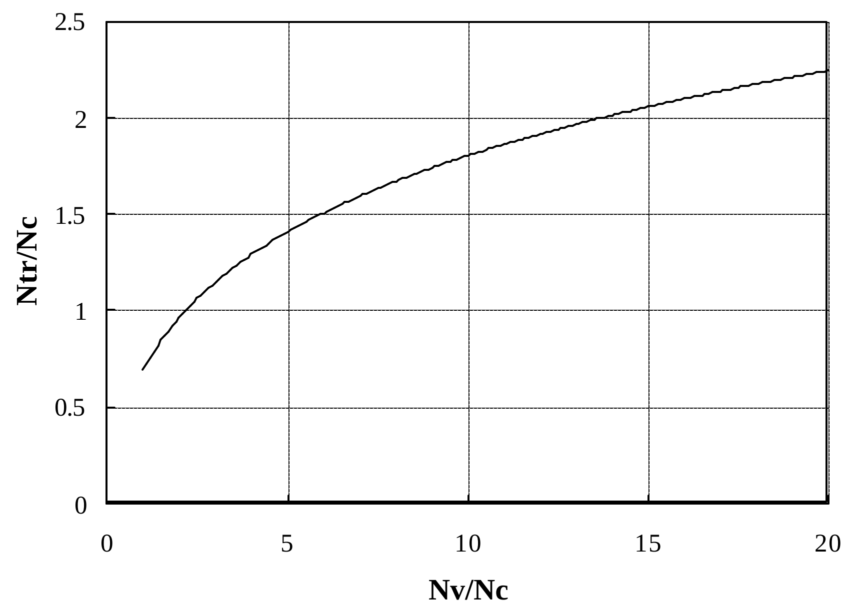

Figure 4.10.4 Normalized transparency carrier density versus the ratio of the effective density of states in the valence and conduction band.

To include multiple hole levels one simply replaces N_v by N_v* as described in section 4.4.3.d.

4.10.2. Principle of operation of a laser diode

A laser diode consists of a cavity, defined as the region between two mirrors with reflectivity R_1 and R_2, and a gain medium, in our case a quantum well. The optical mode originates in spontaneous emission, which is confined to the cavity by the waveguide. This optical mode is amplified by the gain medium and partially reflected by the mirrors. The modal gain depends on the gain of the medium, multiplied with the overlap between the gain medium and the optical mode which we call the confinement factor, Γ, or:

$$\text{modal gain} = g(N)\Gamma \tag{4.10.21}$$

This confinement factor will be calculated in section 6.2.5. Lasing occurs when for light traveling round trip through the cavity the optical gain equals the losses. For a laser with modal gain *g(N)*Γ and wave guide loss α this condition implies:

$$R_1 R_2 \exp[2(g(N)-a)L]=1 \qquad (4.10.22)$$

where L is the length of the cavity. The distributed loss of the mirrors is therefore:

$$\text{mirror loss} = \frac{1}{L}\ln\frac{1}{\sqrt{R_1 R_2}} \qquad (4.10.23)$$

4.10.3. Longitudinal modes in the laser cavity.

Longitudinal modes in the laser cavity correspond to standing waves between the mirrors. If we assume total reflection at the mirrors this wave contains $N/2$ periods where N is an integer. For a given wave length l and a corresponding effective index, n_{eff}, this yields:

$$N = \frac{2n_{eff}L}{l} \qquad (4.10.24)$$

Because of dispersion in the waveguide, a second order model should also include the wavelength dependence of the effective index. Ignoring dispersion we find the difference in wavelength between two adjacent longitudinal modes from:

$$N = \frac{2n_{eff}L}{l_1} \qquad (4.10.25)$$

$$N+1 = \frac{2n_{eff}L}{l_2} \qquad (4.10.26)$$

$$\Delta l = 2Ln_{eff}\left(\frac{1}{N}-\frac{1}{N+1}\right) \cong \frac{l_1^2}{2Ln_{eff}} \qquad (4.10.27)$$

Longer cavities therefore have closer spaced longitudinal modes. An edge emitting (long) cavity with length of 300 μm, n_{eff} = 3.3, and l = 0.8 μm has a wavelength spacing Δl of 0.32 nm while a surface emitting (short) cavity of 3 μm has a wavelength spacing of only 32 nm. These wavelength differences can be converted to energy differences using:

$$\Delta E = \frac{hc}{2Ln_{eff}} = E_{ph}\frac{\Delta l}{l_2} \qquad (4.10.28)$$

so that 0.32 nm corresponds to -6.2 meV and 32 nm to 620 meV. A typical width of the optical gain spectrum is 60 meV, so that an edge emitter biased below threshold can easily contain 10 longitudinal modes, while for a surface emitter the cavity must be carefully designed so that the longitudinal mode overlaps with the gain spectrum.

A more detailed analysis of a Fabry-Perot etalon is described in A.7.3, providing the reflectivity, absorption and transmission as a function of photon energy.

4.10.4. Waveguide modes[3]

The optical modes in the waveguide determine the effective index used to calculate the longitudinal modes as well as the confinement factor which affects the modal gain. Starting from Maxwell's equations in the absence of sources:

$$\vec{\nabla}\times\vec{H} = e_0 n^2(x,y,z) \tag{4.10.29}$$

$$\vec{\nabla}\times\vec{E} = m_0 \frac{\partial\vec{H}}{\partial t} \tag{4.10.30}$$

and assuming a propagating wave in the z-direction and no variation in the y-direction we obtain the following one-dimensional reduced wave equation for a time harmonic field, $E = E_x\, e^{jwt}$, of a TM mode:

$$\frac{d^2E_x}{dx^2} + (n^2(x)k^2 - b^2)E_x = 0 \tag{4.10.31}$$

with the propagation constant given by $b = \frac{w}{c} n_{eff}$, and $k = \frac{w}{c}$, this equation becomes:

$$\frac{d^2E_x}{dx^2} + (n^2(x) - n_{eff}^2)E_x = 0 \tag{4.10.32}$$

this equation is very similar to the Schrödinger equation. In fact previous solutions for quantum wells can be used to solve Maxwell's equation by setting the potential $V(x)$ equal to $-n^2(x)$ and replacing $\frac{w^2}{c^2}$ by $\frac{2m^*}{\hbar^2}$. The energy eigenvalues, E_l, can then be interpreted as minus the effective indices of the modes: $-n^2_{\mathrm{eff},l}$. One particular waveguide of interest is a slab waveguide consisting of a piece of high refractive index material, n_1, with thickness d, between two infinitely wide cladding layers consisting of lower refractive index material, n_2. From Appendix A.1.3. one finds that only one mode exists for:

$$V_0 = -n_2^2 + n_1^2 \quad E_{10} = \frac{c^2}{w^2}\frac{p^2}{d^2} \tag{4.10.33}$$

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[3] A detailed description of modes in dielectric waveguides can be found in Marcuse, "dielectric waveguides", 2nd ed.

or,

$$d \le \frac{c}{w} p \frac{1}{\sqrt{n_1^2 - n_2^2}} = \frac{l}{2\sqrt{n_1^2 - n_2^2}} \tag{4.10.34}$$

For $\lambda = 0.8$ µm, $n_1 = 3.5$ and $n_2 = 3.3$ one finds $d \le 0.34$ µm.

4.10.5. The confinement factor

The confinement factor is defined as the ratio of the modal gain to the gain in the active medium at the wavelength of interest:

$$\Gamma = \frac{\text{modal gain}}{g} = \frac{\int_{-\infty}^{\infty} g(x)|E_x|^2 dx}{\int_{-\infty}^{\infty} |E_x|^2 dx} \tag{4.10.35}$$

for a quantum well with width L_x, the confinement factor reduces to

$$\Gamma = \frac{\int_{-L_x/2}^{L_x/2} |E_x|^2 dx}{\int_{-\infty}^{\infty} |E_x|^2 dx} \tag{4.10.36}$$

$\cong$ 0.02...0.04 for a typical GaAs single quantum well laser

4.10.6. The rate equations for a laser diode.

Rate equations for each longitudinal mode, λ, with photon density S_λ and carrier density N_λ which couple into this mode are:

$$\frac{\partial N_l}{\partial t} = \frac{J_l}{q} - B_l N_l^2 - \frac{N_l}{2t_0} + \sum_k \frac{N_l}{t_{k,l}} - \sum_k \frac{N_l}{t_{kl,k}} - \frac{\partial S_l}{\partial x}\frac{\partial x}{\partial t} \tag{4.10.37}$$

$$\frac{\partial S_l}{\partial t} = b_l B_l N_l^2 - \frac{S_l}{t_{ph,l}} + \frac{\partial S_l}{\partial x}\frac{\partial x}{\partial t}, \lambda = 1, 2, ..., \lambda_{MAX} \tag{4.10.38}$$

Rather than using this set of differential equations for all waveguide modes, we will only consider one mode with photon density S, whose photon energy is closest to the gain peak. The intensity of this mode will grow faster than all others and eventually dominate. This simplification avoids the problem of finding the parameters and coefficients for every single mode. On the other hand it does not enable to calculate the emission spectrum of the laser diode. For a single longitudinal mode the rate equations reduce to:

$$\frac{dN}{dt} = \frac{J}{q} - BN^2 - \frac{N}{2t_0} - v_{gr}\Gamma l(N - N_{tr})S \tag{4.10.39}$$

$$\frac{dS}{dt} = bBN^2 - \frac{S}{t_{ph}} + v_{gr}\Gamma l(N - N_{tr})S \tag{4.10.40}$$

$$P_1 = v_{gr}SW \ln\frac{1}{\sqrt{R_1}} \tag{4.10.41}$$

4.10.6.1. DC solution to the rate equations

The time independent rate equations, ignoring spontaneous emission are:

$$0 = \frac{J_0}{q} - BN_0^2 - \frac{N_0}{2t_0} - v_{gr}\Gamma l(N_0 - N_{tr})S_0 \tag{4.10.42}$$

$$0 = -\frac{S_0}{t_{ph}} + v_{gr}\Gamma l(N_0 - N_{tr})S_0 \tag{4.10.43}$$

where the photon life time is given by:

$$\frac{1}{t_{ph}} = \frac{1}{S}\frac{\partial x}{\partial t}\frac{\partial S}{\partial x} = v_{gr}(a + \frac{1}{L}\ln\frac{1}{\sqrt{R_1 R_2}}) \tag{4.10.44}$$

from which we can solve the carrier concentration while lasing:

$$N_0 = N_{tr} + \frac{1}{t_{ph}v_{gr}\Gamma l} \tag{4.10.45}$$

which is independent of the photon density[4]. The threshold current density is obtained when $S_0 = 0$

$$J_0\big|_{(S_0=0)} = J_{th} = q(BN_0^2 + \frac{N_0}{2t_0}) \tag{4.10.46}$$

The photon density above lasing threshold, and power emitted through mirror R_1, are given by:

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[4] a more rigorous analysis including gain saturation reveals that the carrier concentration does increase with increasing current, even above lasing. However this effect tends to be small in most laser diodes.

$$S_0 = \frac{J_0 - J_{th}}{q} \frac{1}{v_{gr}\Gamma(N_0 - N_{tr})} \tag{4.10.47}$$

and the power emitted through mirror 1 is:

$$P_{1,0} = hnS_0 W v_{gr} \ln \frac{1}{\sqrt{R_1}} \tag{4.10.48}$$

The differential efficiency of the laser diode is:

$$\text{D.E.} = \frac{dP_{1,0}}{dI_0} = \frac{hn}{q} \frac{\ln \frac{1}{\sqrt{R_1}}}{\ln \frac{1}{\sqrt{R_1 + R_2}} + aL} \tag{4.10.49}$$

and the quantum efficiency is:

$$h = \frac{q}{hn} \frac{dP_{1,0}}{dI_0} = \frac{\ln \frac{1}{\sqrt{R_1}}}{\ln \frac{1}{\sqrt{R_1 + R_2}} + aL} \tag{4.10.50}$$

Efficient lasers are therefore obtained by reducing the waveguide losses, increasing the reflectivity of the back mirror, decreasing the reflectivity of the front mirror and decreasing the length of the cavity. Decreasing the reflectivity of the mirror also increases the threshold current and is therefore less desirable. Decreasing the cavity length at first decreases the threshold current but then rapidly increases the threshold current.

4.10.6.2. AC solution to the rate equations

Assuming a time-harmonic solution and ignoring higher order terms (as we did for the LED) the rate equations become:

$$jwn_1 = \frac{j_1}{q} - \frac{n_1}{t_{eff}} - v_{gr}\Gamma l(N_0 - N_{tr})s_1 - v_{gr}\Gamma l n_1 S_0 \tag{4.10.51}$$

$$jws_1 = -\frac{s_1}{t_{ph}} + v_{gr}\Gamma l(N_0 - N_{tr})s_1 + v_{gr}\Gamma l n_1 S_0 \tag{4.10.52}$$

where t_{eff} is the same as for an LED and given by equation [6.1.19]. Using $v_{gr}\Gamma l(N_0 - N_{tr}) = \frac{1}{t_{ph}}$ these equations can be solved yielding:

$$j_1 = jwqn_1 + qn_1(\frac{1}{t_{eff}} + \Gamma l S_0 v_{gr}) + qn_1 \frac{\Gamma l S_0 v_{gr}}{jwt_{ph}} \qquad (4.10.53)$$

replacing n_1 by relating it to the small signal voltage v_1

$$v_1 = \frac{mV_t n_1}{N_0} \qquad (4.10.54)$$

The equation for the small signal current, i_1, can be written as

$$i_1 = (jwC + \frac{1}{R} + \frac{1}{jwL})v_1 \qquad (4.10.55)$$

with $C = \frac{qN_0 A}{mV_t}$, and $m = \frac{N_0 e^{N/N_c}}{N_c(e^{N/N_c} - 1)} + \frac{N_0 e^{N/N_v}}{N_v(e^{N/N_v} - 1)}$, where A is the area of the laser diode.

$$\frac{1}{R} = C(\frac{1}{t_{eff}} + \Gamma l S_0 v_{gr}) \qquad (4.10.56)$$

and

$$L = \frac{1}{C}\frac{t_{ph}}{\Gamma l S_0 v_{gr}} \qquad (4.10.57)$$

4.10.6.3. Small signal equivalent circuit

Adding parasitic elements and the circuit described by the equation [6.2.48] we obtain the following equivalent circuit, where L_B is a series inductance, primarily due to the bond wire, R_s is the series resistance in the device and C_p is the parallel capacitance due to the laser contact and bonding pad.

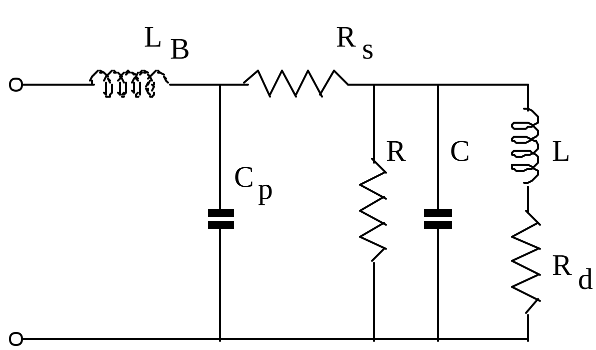

Figure 4.10.5 Small signal equivalent circuit of a laser diode

The resistor, R_d, in series with the inductor, L, is due to gain saturation[5] and can be obtained by adding a gain saturation term to equation [6.2.16]. The optical output power is proportional to the current through inductor L, $i_{1,L}$, which is given by:

$$i_{1,L} = \frac{qAs_1}{t_{ph}} = qAs_1 v_{gr} (a + \frac{1}{L} \ln \frac{1}{\sqrt{R_1 + R_2}}) \qquad (4.10.58)$$

and the corresponding power emitted from mirror R_1

$$p_1 = s_1 hn v_{gr} W \ln \frac{1}{\sqrt{R_1}} \qquad (4.10.59)$$

Ignoring the parasitic elements and the gain saturation resistance, R_d, one finds the ac responsivity p_1/i_1 as:

$$\frac{p_1}{i_1} = \frac{hn}{q} \frac{\ln \frac{1}{\sqrt{R_1}}}{\ln \frac{1}{\sqrt{R_1 + R_2}} + aL} \frac{1}{1 + jw \frac{L}{R} + (jw)^2 LC} \qquad (4.10.60)$$

from which we find the relaxation frequency of the laser:

$$w_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{\Gamma l S_0 v_{gr}}{t_{ph}}} = \sqrt{\frac{\Gamma l P_0}{t_{ph} hn W \ln \frac{1}{\sqrt{R_1}}}} \qquad (4.10.61)$$

or the relaxation frequency is proportional to the square root of the DC output power. The amplitude at the relaxation frequency relative to that at zero frequency equals:

$$\frac{p_1|_{w=w_0}}{p_1|_{w=0}} = \frac{R}{Lw_0} = \frac{1}{\frac{1}{w_0 t_{eff}} + t_{ph} w_0} \qquad (4.10.62)$$

4.10.7. Threshold current of multi-quantum well laser

Comparing threshold currents of laser diodes with identical dimensions and material parameters but with a different number of quantum wells, m, one finds that the threshold currents are not simple multiples of that of a single quantum well laser.

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[5] for a more detailed equivalent circuit including gain saturation see: Ch. S. Harder et al. High-speed GaAs/AlGaAs optoelectronic devices for computer applications, IBM J. Res. Develop., Vol 34, No. 4, July 1990, p. 568-584.

Let us assume that the modal gain, g, is linearly proportional to the carrier concentration in the wells and that the carriers are equally distributed between the m wells. For m quantum wells the modal gain can be expressed as:

$$g = lm(N - N_{tr}) = l\Delta Nm \tag{4.10.63}$$

where l is the differential gain coefficient and N_{tr} is the transparency carrier density. Since the total modal gain is independent of the number of quantum wells we can express the carrier density as a function modal gain at lasing[6].

$$N = \frac{g}{lm} + N_0 = \frac{\Delta N}{m} \tag{4.10.64}$$

The radiative recombination current at threshold is then

$$J_{th} = qB_1 m(N_{tr} + \frac{\Delta N}{m})^2 = qB_1(N_{tr}^2 m + 2N_{tr}\Delta N + \frac{\Delta N^2}{m}) \tag{4.10.65}$$

This means that the threshold current density is a constant plus a component, which is proportional to the number of quantum wells. The last term can be ignored for $m >> 1$ and $\Delta N << N_{tr}$.

4.10.8. Large signal switching of a laser diode

Because of the non-linear terms in the rate equations the large signal switching of a laser diode exhibits some peculiar characteristics. The response to a current step is shown in the figure below. The carrier density initially increases linearly with time while the photon density remains very small since stimulated emission only kicks in for $N > N_0$.

[6] We assume here that we are comparing identical lasers, which only differ by the number of quantum wells.

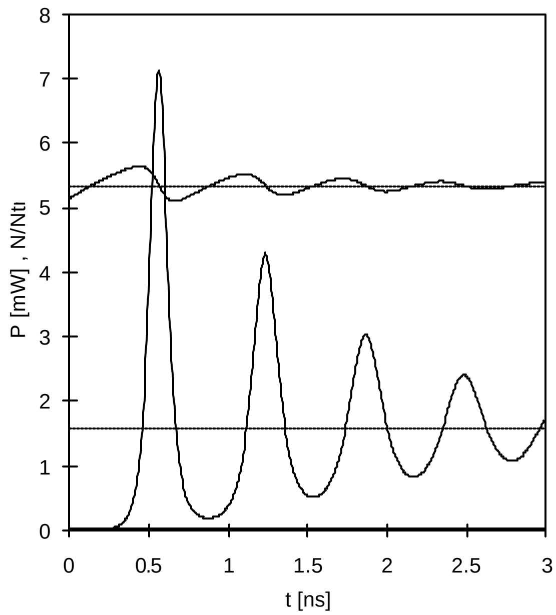

Figure 4.10.6 Optical power and normalized carrier concentration versus time when applying a step current at $t = 0$ from $I = 0.95\ I_{th}$ to $I = 1.3\ I_{th}$.

Both the carrier density and the photon density oscillate around their final value. The oscillation peaks are spaced by roughly $2\pi/\omega_0$, where ω_0 is the small signal relaxation frequency at the final current. The photon and carrier densities are out of phase as carriers are converted into photons due to stimulated emission, while photons are converted back into electron-hole pairs due to absorption. High-speed operation is obtained by biasing close to the threshold current and driving the laser well above the threshold. In addition one can use the non-linear behavior to generate short optical pulses. By applying a current pulse, which is long enough to initiate the first peak in the oscillation, but short enough to avoid the second peak, one obtains an optical pulse which is significantly shorter that the applied current pulse. This method is referred to as gain switching or current spiking.