Example 4.3 Consider an abrupt p-n diode with $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. Calculate the junction capacitance at zero bias. The diode area equals 10^{-4} cm^2. Repeat the problem while treating the diode as a one-sided diode and calculate the relative error.

Solution The built-in potential of the diode equals:

$$\phi_i = V_t \ln \frac{N_d N_a}{n_i^2} = 0.83 \text{ V}$$

The depletion layer width at zero bias equals:

$$x_d = \sqrt{\frac{2\varepsilon_s (\phi_i - V_a)}{q} \left(\frac{1}{N_a} + \frac{1}{N_d}\right)} = 0.33 \ \mu\text{m}$$

And the junction capacitance at zero bias equals:

$$C_{j0} = A \frac{\varepsilon_s}{x_d}\bigg|_{V_a = 0} = 3.17 \text{ pF}$$

Repeating the analysis while treating the diode as a one-sided diode, one only has to consider the region with the lower doping density so that:

$$x_d \cong x_n = \sqrt{\frac{2\varepsilon_s}{qN_d} (\phi_i - V_a)} = 0.31 \, \mu\text{m}$$

Note that N_a was still needed to calculate ϕ_i.
The junction capacitance at zero bias equals:

$$C_{j0} = A \frac{\varepsilon_s}{x_d}\bigg|_{V_a = 0} = 3.18 \text{ pF}$$

The relative error equals 0.5 %, which justifies the use of the one-sided approximation.